980

SPECULATIONS ON THE FOURTH DIMENSION

SELECTED WRITINGS OF CHARLES H. HINTON

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CONTENTS

Introduction	v
What Is the Fourth Dimension?	1
A Plane World	23
A Picture of Our Universe	41
Casting Out the Self	56
Many Dimensions	67
An Unfinished Communication	80
A New Era of Thought	106
The Fourth Dimension	120
The Recognition of the Fourth Dimension	142
An Episode of Flatland	163

INTRODUCTION

The father James Hinton was an ear surgeon who was best known for *The Mystery of Pain,* a little book which sets forth the Panglossian thesis that "all that which we feel as painful is really *giving*—something that our fellows are better for, even though we cannot trace it." It gives some idea of the turn of the son Charles Hinton's mind to learn that he wrote a piece, "The Persian King," in which he attempted to use higher dimensions and infinite series to obtain a mathematically accurate model of this idea.

Charles Howard Hinton was a professional mathematician—he took the master's degree at Oxford, taught at Princeton, and published pure mathematics related to work of Morley, Hamilton, and Cayley—but for him formal mathematics was never an end in itself.

Hinton's touchstone was, rather, direct and intuitive knowledge of four-dimensional space. The bulk of his writings are aimed at developing in the reader the power to think about 4-D space; and the rest of his work focuses on using a knowledge of higher space to solve various problems in physics and in metaphysics.

Hinton was born in London in 1853, the first son in his family. He was schooled at Rugby, and matriculated at Oxford in 1871. From a letter written to him by his father in 1869, we learn that already while at Rugby, Hinton evidenced an interest in "studying geometry as an exercise of direct perception."

After two years at Oxford, he was granted a three-year term as Exhibitioner of Balliol College in the University. This meant that the college paid him a stipend or "exhibition" to pursue his studies there. On the strength of this honor, Hinton obtained a post as Assistant Master at the Cheltenham Ladies' College in 1875. He continued his studies at Oxford, some 50 miles distant from Cheltenham, receiving his B.A. in 1877. In 1880, he left Cheltenham to teach at the Uppingham School, where he remained until he received his M.A. from Oxford in 1886. It is during the decade 1877–1887 that Hinton found his life work.

Of this period he writes in *A New Era of Thought* that, "I found myself in respect to knowledge like a man who is in the midst of plenty and yet who cannot find anything to eat. All around me were the evidences of knowledge—the arts, the sciences, interesting talk, useful inventions—and yet I myself was profited nothing at all; for somehow, amidst all this activity, I was left alone, I could get nothing which I could know." Desperate for some absolute knowledge, Hinton hit upon the plan of memorizing a cubic yard of one-inch cubes. That is, he took a 36 × 36 × 36 block of cubes, assigned a two-word Latin name (e.g. *Collis Nebula*) to each of the 46,656 units, and learned to use this network as a sort of "solid paper." Thus when he wished to visualize some solid structure, he would do so by adjusting its size so that it fit into his cubic yard. Then he could describe the structure by listing the names of the occupied cells. Hinton maintains that he thereby obtained a sort of direct and sensuous appreciation of space.

Given that Hinton's father had been known for his exceptional memory, and that there is a system which reduces the brute facts to be memorized to 216, this learning of a block of one-inch cubes is not inconceivable. But now Hinton went on to memorize the positions of the little cubes for each of the 24 possible orientations (six choices for the bottom face times four choices for the front face) which the block might have relative to the observer.

His reasons for doing this are described in his essay "Casting Out the Self." If cube A is touching cube B, this is an absolute fact. But to say that cube A is above or behind cube B is simply to say something about the relation of the self to the arrangement of cubes. It was in order to eliminate such "self-elements" that Hinton learned the block of cubes in each of its 24 possible orientations.

This casting out of self-elements led to an interesting question: Is the difference between an arrangement of cubes and the mirror image of this arrangement absolute or relative? Hinton brooded over Kant's remarks (in section 13 of the *Prolegomena*) on the question of whether the apparently irreconcilable difference between a right hand and a left hand is not somehow the result of a limited space intuition; and he may have heard of A. F. Möbius's 1827 discovery that any 3-D object can be turned into its mirror-image by a rotation through 4-D space.

In any case, Hinton now became interested in the fourth dimension. He used his "solid paper" to construct for himself the various cubical cross-sections of the hypercube or tesseract (a word which Hinton may have coined himself), assigning a different color to each of the 81 parts

(1 tesseract + 8 cubes + 24 squares + 32 line segments + 16 points) of the tesseract to keep things straight. By working with these cross-sections he was able to visualize the reality of the fact that if a tesseract is pushed through our space, turned over, and pushed back through, then the last cubical cross-section seen will be the mirror image of the last seen the first time through.

As his understanding of the fourth dimension grew, Hinton set to writing about it. His first published essay, "What Is the Fourth Dimension?," appeared in 1880 in the *Dublin University Magazine,* was reprinted in the *Cheltenham Ladies' College Magazine* of September 1883, and finally was published as a pamphlet, with the subtitle *Ghosts Explained,* by Swann Sonnenschein & Co. in 1884. In the period 1884–1886, Swann Sonnenschein published in London nine different such pamphlets by Hinton which were then collected in the two-volume set, *Scientific Romances.*

Swann Sonnenschein was to be the original publisher for all of Hinton's books. Around 1910, Allen & Unwin obtained the rights to the books, which they kept in print for a number of years. In the introduction to *The Fourth Dimension* Hinton acknowledges his debt to "the publisher of this volume, Mr. Sonnenschein, to whose unique appreciation of the line of thought of this, as of my former essays, their publication is owing."

All but three of the *Scientific Romances* are partially reprinted below. I have omitted the lengthy "The Persian King," subtitled "The Mystery of Pleasure and Pain," which is about a king who keeps a valley running by absorbing a small amount of pain from everyone, so that there is enough differential between pleasure and pain for activity to exist. I have left out "On the Education of the Imagination," which is basically an exhortation to memorize large blocks of cubes; and I have left out the long novella *Stella.*

This last piece is a first-person description of the narrator's love affair with a girl who has been made invisible by her guardian. That is, he has provided her with an elixir which she drinks to make her index of refraction equal to that of air. This work can be viewed as a very early piece of science fiction (perhaps providing the inspiration for H. G. Wells's *The Invisible Man*), but it also has a certain didactic purpose. Hinton shared with his father a very strong conviction that the key to right living is openness and altruism. As Stella says, "Being is being for others." The reason Stella chooses to remain invisible is so that she will not fall into a self-serving concern with her own ap-

pearance. Hinton clearly was groping for some higher-dimensional justification of his ethical beliefs in *Stella,* but the time was not yet ripe.

The most interesting idea in *Stella,* that of two-dimensional time, is treated in a more vivid fashion in the stunning tale "An Unfinished Communication." Two-dimensional time works something like this: First start with the idea of your entire life as being a fixed object in 4-D spacetime, then imagine that while some second time lapses, your entire life gradually evolves into a different one. Hinton returns to this theme in *A New Era of Thought,* where, rather than suggesting a second time dimension, he employs the notion of the eternal return and speaks of life as a sort of phonograph record which is played over and over, undergoing slight alterations each time.

Returning to the *Scientific Romances* collection, we have "A Plane World," a cousin of Abbott's 1884 *Flatland,* which Hinton refers to in a brief introduction to his essay: "And I should have wished to be able to refer the reader altogether to that ingenious work, 'Flatland'. But on turning over its pages again, I find that the author has used his rare talent for a purpose foreign to the intent of our work. For evidently the physical conditions of life on the plane have not been his main object. He has used them as a setting wherein to place his satire and his lessons. But we wish, in the first place, to know the physical facts."

A really remarkable feature of the 2-D world described in "A Plane World" is that it exists on the surface of a sphere, instead of a plane; and is thus more similar to Dionys Burger's Sphereland than to Edwin A. Abbott's Flatland. The logical step to take after describing this 2-D world on a sphere would be to suggest that our 3-D space is the hypersurface of a hypersphere. Bernhard Riemann hinted at this idea as early as 1854, but it was Einstein who first seriously advocated it.

"A Picture of Our Universe" was written as a companion piece to "A Plane World," and here Hinton makes a sustained effort to find some physical application for the notion of 4-D space. Hinton's working hypothesis, which he presented as early as the end of "What Is the Fourth Dimension?," is that the matter of our world actually has a slight hyperthickness in the direction of a fourth spatial dimension.

In the first part of the essay under discussion, Hinton attempts to explain static electricity as a twisting of matter in 4-D space. He comes remarkably close here to anticipating the modern notion of antimatter—as Martin Gardner has pointed out in Chapter 24 of *The*

Ambidextrous Universe, which also contains a reference to the Kaluza-Klein five-dimensional theory of relativity which in some sense formalizes the idea Hinton presented here. In a much later essay, "The Recognition of the Fourth Dimension," Hinton was to present a very different 4-D modeling of static electricity.

In the second part of "A Picture of Our Universe," Hinton describes his conception of the aether as a 4-D hypersolid upon which the (hyperthin) 4-D objects of our space move about. This represents a distinct deviation from modern physical theory. As Hinton says in *A New Era of Thought,* "We will suppose, then, that we are not *in,* but *on* the aether, only not on it in any known direction, but that the new direction is that which comes in. The aether is a smooth body, along which we slide, being distant from it at every point about the thickness of an atom." [My italics.] Now, the question of whether or not our matter is ultimately four-dimensional is, to say the least, open. But there are now no scientists who would assent to the conception of a fixed spatial aether *next to* our universe.

If there is an aether, it is to be conceived of as something which we are *in*—primarily because only needless complications result if anything else is assumed. Indeed, it might not be too much to say that Hinton's belief that we are not in, but on the aether proved to be the major stumbling block in his attempts to attain a higher-dimensional physics. It should be pointed out on Hinton's behalf that he had good reason for believing that we cannot be *in* the luminiferous aether. The conclusion from the observed properties of light and the physics of the time seemed to be that the aether is solid—and we cannot very well be moving around inside a solid.

This whole puzzle was dissolved by the Special Theory of Relativity in 1905 with the abolishing of *space* aether in favor of *spacetime* aether. It is hard, initially, to appreciate the difference. But a curious suggestion which Hinton makes at the end of "A Picture of Our Universe" (and which he returns to in *An Episode of Flatland*) serves to clarify the issue. Hinton suggests that, "A being able to lay hold of the aether by any means would, unless he were instantly lost from amongst us by his staying still while the earth dashes on—he would be able to pass in any space direction in our world. He would not need to climb by stairs, nor to pass along resting on the ground." In other words, if one only had some 4-D pitons, then one could climb up the face of the aether to float a hundred feet above the ground. But a marker driven into the aether would supply an absolute standard of

rest—which is ruled impossible by Special Relativity. However, as Einstein pointed out in his essay "Ether and the Theory of Relativity," the very notion of the spacetime metric tensor's existence in empty spacetime serves to validate the notion of a *spacetime* aether.

The only selection from *Scientific Romances* I will not discuss at length is the piece "Many Dimensions." This is one of the most inspiring of Hinton's writings. With his usual elegance of style he moves from the question of "Why not *five* dimensions?" to drawing ethical lessons from the existence of 4-D space.

Soon after Hinton recieved his master's degree from Oxford in 1886, he left his post at Uppingham School to teach for several years in the Japanese government middle schools. He left the book *A New Era of Thought* in manuscript form with his friends Alicia Boole and H. John Falk. They used his manuscript, essentially unrevised, for Part 1 of *A New Era of Thought*; and for Part 2 they themselves wrote up a detailed description of the exercises with colored cubes which Hinton had employed to get a mental image of the tesseract.

Part 1 consists of a somewhat rambling and disconnected series of chapters about the possibility of thinking four-dimensionally, and about the religious and philosophical insights thus obtainable. This part bears evidence of not having been reworked by the author; for instance, Hinton somehow neglects to make a crucial point (that a cube can be turned into its mirror image by rotation through the fourth dimension) which he spends several chapters leading up to. Nevertheless, this book was written at the high point of Hinton's confidence in the fourth dimension as a sort of philosopher's stone for human inquiry, and his enthusiasm is infectious.

Chapters 7 and 9 of *A New Era of Thought* (the sections entitled "Self Elements in Our Consciousness" and "Another View of the Aether" in the present volume) are of particular interest. Some treatments of the fourth dimension, for example that of E. H. Neville in his *The Fourth Dimension,* proceed purely analytically, treating 4-D space as nothing but a collection of ordered quadruples of real numbers with certain "distance" and "angle" relations defined among these quadruples. Other treatments, for example that of H. P. Manning in his *Geometry of Four Dimensions,* proceed synthetically in the manner of Euclid, stating axioms about 4-D entities from which theorems are then derived. Hinton is one of the very few who has held to the intuitive and descriptive approach in discussing 4-D space. In the section of *A New Era of Thought* entitled "Self Elements in Our Consciousness," he provides a very ingenious rebuttal to the so-com-

monly-heard philistinism that it is hopeless to imagine what the extra spatial dimension looks like because no one has ever seen it.

In Chapter 9 Hinton describes in some detail his phonograph-record view of the aether. The idea is that the aether has grooves which control the behavior of the matter sliding over it. The assumption seems to be that every so often the Earth will slide back to its starting point on the aether, and then history will repeat itself. In order to make room for free will and the sort of 2-D time he describes in "An Unfinished Communication," Hinton assumes that each time the eternal return is accomplished and we slide over the grooves of our life again, we are able to alter slightly the directions of the grooves. He goes on to draw the moral that since we all have aethereal bodies which are grooves on the one aether, then the mystic teaching All is One is to be upheld.

Hinton's father James once had his marriage proposal rejected because he was suspected of having abandoned Christianity for the abstract ideal of altruism. (The third time he asked the girl, a Miss Haddon, she accepted.) Hinton proves himself a loyal son in this book, written some ten years after the father's death from inflammation of the brain in 1875. There is a great deal in Part 1 about the idea that the sooner we start thinking of those around us as the same as ourselves, the sooner we will be able to enter into the higher mysteries.

The second part of *A New Era of Thought* consists of a description of how to visualize a tesseract by looking at various 3-D cross sections of it. One is to construct a set of 12 cubes, coloring the faces, edges and corners all manner of different colors. (Eighty-one different colors are used, and some rather unfamiliar ones are resorted to. The modern reader may be amused to note that the line going into the fourth dimension relative to the first cube is to be colored "stone.") Eight of these cubes make up the boundaries of the hypercube, and the four others are cross sections taken between pairs of opposite cubes. The way in which all the cubes fit together is really explained rather well, if one has the will to endure not only 81 colors, but the 81 Latin names which Hinton assigns to the parts of the tesseract.

In addition to the set of 12 large cubes, there was also to be a set of 81 small monochrome cubes, each representing a part of the tesseract. By moving these little cubes about one could better comprehend the fact that rotation through the fourth dimension corresponds to mirror image reflection in 3-D space.

Judging from the quotes which Sonnenschein & Co. reprint on the endpapers of Hinton's books, both the *Scientific Romances* and *A*

New Era of Thought were quite favorably received in England. Of the Scientific Romance "What Is the Fourth Dimension?," the *Pall Mall Gazette* wrote, "It is a short treatise of admirable clearness. Mr. Hinton brings us panting, but delighted, to at least a momentary faith in the Fourth Dimension, and upon the eye there opens a vista of interesting problems. It exhibits a boldness of speculation and a power of conceiving and expressing even the inconceivable, which rouses one's faculties like a tonic." The *Literary World* discussed *A New Era of Thought* in equally athletic terms, "A theoretical and practical treatise on the Fourth Dimension. The book is a powerful mental gymnastic; the style is as clear as it can be. The author is in grim earnest, and promises a complete system of four-dimensional thought—Mechanical, Scientific, and Aesthetic."

But meanwhile Hinton was in Japan, perhaps at a position where he was allowed to drill students in the use of his cubes. Such far-ranging travels were nothing unusual for Hinton's family. His grandfather, a Baptist minister, had been across the Atlantic and was best known for his book *The History and Topography of the United States of America*. His father spent some time working on a ship transporting Blacks from Sierra Leone to Jamaica, lived for a while in New Orleans, and owned property in the Azores, where he died.

Exactly how long Hinton stayed in Japan is unclear, but in the Fall of 1893 he started a stint as Instructor in the Mathematics Department of Princeton University. There were only three other teachers in the department at that time, but Hinton seems to have been at the bottom of the pecking order. He taught only Freshman and Sophomore level courses—Elementary Solid Geometry and Mensuration, Trigonometry, and Conic Sections, in particular—and he was fired after four years. In a piece called "The Oxford Spirit" which he wrote for the *Independent* in 1902, Hinton expressed some bitterness at his treatment by the American educational system. "In America a new phenomenon has arisen—the business man in control of the halls of learning. The college president runs his university as if it were a great factory. He makes a number of provisional appointments—instructors. The instructors know that during term time they must work to the full extent of their energy in instruction, and occupy their vacations in prosecuting their subject if they are to retain their positions."

It cannot really be said that Hinton followed this formula while teaching at Princeton. He was friendly with his students and became very interested in baseball—so much so that he invented a sort of base-

ball gun. As he wrote in his article "A Mechanical Pitcher," in *Harper's Weekly,* March 20, 1897, "The project of constructing a mechanical pitcher was suggested to me by considerations all who are interested in baseball will appreciate. I had remarked the frequent occurence of 'sore arm', as also that it was only in the matches themselves that many a batter had experience of really first class pitching." So, after a period of experimentation, he developed a gun which, when charged with gunpowder, would shoot a baseball at the requisite 40 to 70 miles per hour. The speed was adjusted by varying the size of the adjustable breech; and, most importantly, any desired curve or drop ball could be shot by adjusting two rubber-coated steel fingers attached to the muzzle of the gun. The Princeton nine worked out with this mechanical pitcher for several years, although it was eventually abandoned because of the fear it inspired in the batters.

There is a curious story connected with Hinton's discovery of the baseball gun which is reported in his obituary, which Gelett Burgess contributed to the *New York Sun* on May 5, 1907. "Although on account of his enthusiasm for metageometry he was never a great success as an instructor in his college positions, he made many friends, and in Princeton endeared himself to the students by one of the most successful practical jokes ever perpetrated there. This was just after his perfection of the baseball gun. He invited the faculty and students to a lecture, at which he demonstrated the machine and described its scientific theory. While he was upon the platform the lecture was interrupted by the arrival of a special delivery postman who walked down the aisle and called to the professor. As he had been the victim of many practical jokes and 'horsed' by the students according to the Princeton custom, the audience prepared for some absurd diversion. After protesting against the interruption, but not being able to send away the messenger, Prof. Hinton begged permission to look at a letter important enough to demand consideration at such a time. He read aloud, and had turned two pages, reading an account of a baseball game in the year 1950, before the students discovered that the joke was upon them." Presumably this fictional baseball game included the use of Hinton's baseball gun. In the article on his gun mentioned above he indicates that he expected the use of it to become commonplace. There is something magically apt in this story of Hinton, the student of spacetime, seeming to receive a message from the future.

The students' nickname for Hinton at Princeton was "Bull," but it seems that this nickname was intended to have the connotation of

physical strength rather than verbal extravagance. In the same obituary we read that "After a Pennsylvania-Princeton football game, Prof. Hinton became the myth hero of the students by throwing bodily over a fence a husky Pennsylvania man who had attempted to snatch a yellow chrysanthemum from the professor's coat."

After leaving Princeton, Hinton took a position as Assistant Professor at the University of Minnesota. He continued using his baseball gun there, and he also gave a number of extracurricular talks and lectures. One, held at his home on 516 Thirteenth Ave., was entitled "Double Personality—Who Am I?" In 1900 he resigned from the University to take a position at the Naval Observatory in Washington, D.C. The famous astronomer Simon Newcomb had recently retired from his post there, but still took an active part in the affairs of the Observatory. In view of the fact that Newcomb wrote a paper on 4-D rotations in the first volume of the *American Journal of Mathematics,* it seems likely that he helped Hinton obtain his position at the Naval Observatory.

In November 1901, Hinton read a paper called "The Recognition of the Fourth Dimension" before the Philosophical Society of Washington. This paper contains two interesting ideas about electricity. First, Hinton shows how the sort of "double rotation" which is possible in 4-D space might be used to give a geometric model of electrical charge. Second, he formulates the idea that an electric current can be geometrically represented as a vortex sheet whose edge meets the aether along the wire of the circuit. He was very proud of this paper. He wrote a popular article about it called "The Fourth Dimension" for *Harper's Monthly Magazine* of July 1904; he formally developed some of its mathematical suggestions in a paper in the *Proceedings of the Royal Irish Academy* of November 1903; and he included it as the last chapter of his book *The Fourth Dimension.*

The Fourth Dimension was published in 1904. The chapters are largely independent of each other, and one has the impression that they were written at various times over the 16 years since publication of *A New Era of Thought.* The tone of this book is much more subdued than that of the earlier works. Hinton remarks that his earlier systems of naming regions of space "turned out, after giving them a fair trial, to be intolerable." He is no longer absolutely sure that he can teach everyone to see the reality of the fourth dimension, and says, "I do not like to speak positively, for I might occasion a loss of time on the part of others, if, as may very well be, I am mistaken."

Nevertheless, Hinton continues to try to teach the reader to see 4-D space. *The Fourth Dimension* contains a greatly streamlined version of the tesseract section models which Hinton introduced in *A New Era of Thought.* There were actually three parts to the complete set of tesseract models. There was a set of 27 "slabs," actually cardboard squares; a set of 81 one-inch monochrome cubes, each a different color; and a set of 12 multicolored "catalogue cubes," which were depicted in a color plate bound into *The Fourth Dimension.* When the book came out, one could buy a set consisting of the 27 slabs and the 81 little cubes for 16 shillings, or a set consisting of the 12 catalogue cubes for 21 shillings. (The book itself cost 4 shillings and sixpence.) It would be interesting to know how many of these sets were actually sold. Appended to the essay "Hypercubes" in Martin Gardner's book *Mathematical Carnival,* there is a curious letter from a former user of Hinton's cubes who calls them "completely mind-destroying."

In Chapters 1, 4, and 5 (included in the present anthology), there are some interesting remarks on the history of 4-D thinking from Parmenides to Bolyai. Chapter 5 has an amazing anticipation of the Special Theory of Relativity—Hinton here works with a non-standard definition of distance which is identical with the definition of the invariant spacetime interval.

In Chapter 8 we hear once again from the Alicia Boole who helped edit *A New Era of Thought.* This was the daughter of George Boole, the renowned logician, and in this chapter Hinton describes a very pretty chain of reasoning which she developed, leading from a sort of 4-D truth-table to an unexpected gap in logical practice. Hinton's father was a friend of the Boole family, and *The Life and Letters of James Hinton* contains numerous references to them. Hinton courted and married Boole's daughter Mary.

He explained his system to one of Mary's younger sisters, Alicia, and she eventually wrote a number of papers on four-dimensional figures. The middle part of her life was devoted to the raising of her family with a Mr. Stott; but when Alicia Boole grew older she came into contact with H. S. M. Coxeter and took up her work on four-dimensional space again. Coxeter's book *Regular Polytopes* (a Dover reprint) describes her and her work on pp. 258–259. Alicia Boole's geometrical understanding and mathematical precision were in some ways greater than Hinton's.

Mary Boole bore Hinton four sons: Eric, George, Sebastian and William. Like her youngest sister Ethel, who wrote novels under the

name E. L. Voynich, Mary Boole Hinton was literary. She was known in Washington as a lecturer on poetry. I have been unable to learn the date of their marriage, but I would hazard the guess that it took place while Hinton was teaching at Cheltenham.

After spending several years at the Naval Observatory, Hinton took the civil service test for a position in the Patent Office, having studied only one night. He qualified as an examiner of chemical patents, and he remained at this post until his sudden death at the age of 54 on April 30, 1907.

Dramatically enough, Hinton suffered cerebral hemorrhage and dropped dead on the spot while leaving the annual banquet of the Washington, D.C., Society of Philanthropic Inquiry. He was a prominent member of the Society and had wound up the evening by complying with the toastmaster's request for a toast to "female philosophers." His death is described in an article called "Scientist Drops Dead" in the *Washington Post* of Wednesday, May 1, 1907.

At the time of his death, Hinton's last book, *An Episode of Flatland: or How a Plane Folk Discovered the Third Dimension*, was in press; and we can fittingly regard this mature and mellow work as his last testament. Any preconceptions of Hinton as some sort of narrow-minded crank are dispelled by the gentle self-irony of this novel, which features two characters (Hugh Farmer, and the school director in Chapter 16), who seem to be modeled on Hinton himself.

An Episode of Flatland returns to Hinton's notion that the aether is a solid body which is next to us in the direction of the fourth dimension, and that our bodies actually have a slight 4-D hyperthickness. In his novel, the soul is taken to be a very small 4-D entity which directs the 3-D motions of the body—just as the captain of a ship is a small 3-D entity which directs the 2-D motions of the ship. The use to which Hinton puts these ideas in his last book is wholly at odds with Special Relativity, but the trick is so clever that one does not really care.

Hinton seems never to have been involved with the active spiritualist movement of his time, despite the fact that the spirit world is customarily said to be in the fourth dimension. He did have a strong belief in the reality of the fourth dimension, and anticipated the day when this could be scientifically proved. "It will be an exhilarating moment when an investigator comes upon phenomena which show that external nature cannot be explained except by the assumption of a four-dimension space," he wrote in *A New Era of Thought*.

Has this moment arrived? Is there anything in physics, chemistry, biology or psychology to suggest that the fourth dimension is anything more than a useful ideal conception? It is certainly true that the spacetime of modern physics is a sort of 4-D space, but spacetime differs essentially from a pure Euclidean 4-D space in several ways, most essentially in the fact that the invariant metric is *interval* rather than *distance*. (As mentioned above, in Chapter 5 of *The Fourth Dimension* Hinton actually seems to have anticipated this development!)

Is there any evidence for Hinton's belief that the *space* of the world is 4-D? It is certainly true that the most natural way of presenting Einstein's theory of gravitation entails viewing our space as a curved hypersurface in some higher-dimensional space. But General Relativity does not seem to demand any hyperthickness to the space of our world, and it is thus possible to think of that higher-dimensional space as simply a compact way of describing a tensor-valued field which exists in our space and time.

The quantum-mechanical treatment of elementary particles involves viewing the particle as an entity in an infinite-dimensional Hilbert space; but, again, this higher-dimensional space is generally regarded as a mathematical tool with no "real" physical correlate. This could just be because there has been no serious effort to think about Hilbert space in the sort of literal and intuitive fashion which Hinton advocates.

There are also a number of new geometrical treatments of elementary particle phenomena which do seem to demand the real existence of higher spatial dimensions—I am thinking here of Wheeler's geometrodynamics program, particularly his "wormhole" theory of electric charge. So there is some hope of confirmation for Hinton's beliefs in the field of particle physics; although at this point in history it is hard to even guess what the final theory of elementary particles will be.

Another area of physics where there is some possibility of attaining a higher-dimensional space is in Hugh Everett's Many Universe and John A. Wheeler's Superspace interpretations of quantum mechanics. These theories contemplate the idea of a continuum of parallel worlds—an idea which Hinton anticipated with his 2-D time—but they have by no means attained widespread acceptance.

In the end it seems that the best place to look for higher-dimensional space is, as Hinton so often said, in the mind. We still have no

idea of how to axiomatize the logical space in which our mind moves about, but there is every reason to believe that this space is higher-dimensional. It has been my personal experience that Hinton's claim that the mind can move in 4-D space is true, although I cannot say that I find the experience of turning the world into its own mirror image a pleasant one.

We are only at the threshold of a non-reductive theory of consciousness; and any final higher-space expression of higher consciousness is something which is no closer now than it was in Hinton's time. Indeed, it would be folly to expect that everything can be explained along any given lines—the world exists both before and after our attempts to understand it. Hinton eloquently continues this line of thought in "Many Dimensions": "If we want to pass on and on till magnitude and dimensions disappear, is it not done for us already? That reality, where magnitudes and dimensions are not, is simple and about us. For passing thus on and on we lose ourselves, but find the clue again in the apprehension of the simplest acts of human goodness, in the most rudimentary recognition of another human soul wherein is neither magnitude nor dimension, and yet all is real."

R.v.B.R.

Geneseo, N.Y.
July 15, 1977

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WHAT IS THE FOURTH DIMENSION?

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Limitations of Our Existence

At the present time our actions are largely influenced by our theories. We have abandoned the simple and instinctive mode of life of the earlier civilizations for one regulated by the assumptions of our knowledge and supplemented by all the devices of intelligence. In such a state it is possible to conceive that a danger may arise, not only from a want of knowledge and practical skill, but even from the very presence and possession of them in any one department, if there is a lack of information in other departments. If, for instance, with our present knowledge of physical laws and mechanical skill, we were to build houses without regard to the conditions laid down by physiology, we should probably—to suit an apparent convenience—make them perfectly draught-tight, and the best-constructed mansions would be full of suffocating chambers. The knowledge of the construction of the body and the conditions of its health prevent it from suffering injury by the development of our powers over nature.

In no dissimilar way the mental balance is saved from the dangers attending an attention concentrated on the laws of mechanical science by a just consideration of the constitution of the knowing faculty, and the conditions of knowledge. Whatever pursuit we are engaged in, we are acting consciously or unconsciously upon some theory, some view of things. And when the limits of daily routine are continually narrowed by the ever-increasing complication of our civilization, it becomes doubly important that not one only but every kind of thought should be shared in.

There are two ways of passing beyond the domain of practical certainty, and of looking into the vast range of possibility. One is by asking, "What is knowledge? What constitutes experience?" If we adopt this course we are plunged into a sea of speculation. Were it not

that the highest faculties of the mind find therein so ample a range, we should return to the solid ground of facts, with simply a feeling of relief at escaping from so great a confusion and contradictoriness.

The other path which leads us beyond the horizon of actual experience is that of questioning whatever seems arbitrary and irrationally limited in the domain of knowledge. Such a questioning has often been successfully applied in the search for new facts. For a long time four gases were considered incapable of being reduced to the liquid state. It is but lately that a physicist has succeeded in showing that there is no such arbitrary distinction among gases. Recently again the question has been raised, "Is there not a fourth state of matter?" Solid, liquid, and gaseous states are known. Mr. Crookes attempts to demonstrate the existence of a state differing from all of these. It is the object of these pages to show that, by supposing away certain limitations of the fundamental conditions of existence as we know it, a state of being can be conceived with powers far transcending our own. When this is made clear it will not be out of place to investigate what relations would subsist between our mode of existence and that which will be seen to be a possible one.

In the first place, what is the limitation that we must suppose away?

An observer standing in the corner of a room has three directions naturally marked out for him; one is upwards along the line of meeting of the two walls; another is forwards where the floor meets one of the walls; a third is sideways where the floor meets the other wall. He can proceed to any part of the floor of the room by moving first the right distance along one wall, and then by turning at right angles and walking parallel to the other wall. He walks in this case first of all in the direction of one of the straight lines that meet in the corner of the floor, afterwards in the direction of the other. By going more or less in one direction or the other, he can reach any point on the floor, and any movement, however circuitous, can be resolved into simple movements in these two directions.

But by moving in these two directions he is unable to raise himself in the room. If he wished to touch a point in the ceiling, he would have to move in the direction of the line in which the two walls meet. There are three directions then, each at right angles to both the other, and entirely independent of one another. By moving in these three directions or combinations of them, it is possible to arrive at any point in a room. And if we suppose the straight lines which meet in the corner

of the room to be prolonged indefinitely, it would be possible by moving in the direction of those three lines, to arrive at any point in space. Thus in space there are three independent directions, and only three; every other direction is compounded of these three. The question that comes before us then is this. "Why should there be three and only three directions?"[1] Space, as we know it, is subject to a limitation.

In order to obtain an adequate conception of what this limitation is, it is necessary to first imagine beings existing in a space more limited than that in which we move. Thus we may conceive a being who has been throughout all the range of his experience confined to a single straight line. Such a being would know what it was to move to and fro, but no more. The whole of space would be to him but the extension in both directions of the straight line to an infinite distance. It is evident that two such creatures could never pass one another. We can conceive their coming out of the straight line and entering it again, but they having moved always in one straight line, would have no conception of any other direction of motion by which such a result could be effected. The only shape which could exist in a one-dimensional existence of this kind would be a finite straight line. There would be no difference in the shapes of figures; all that could exist would simply be longer or shorter straight lines.

Again, to go a step higher in the domain of a conceivable existence. Suppose a being confined to a plane superficies, and throughout all the range of its experience never to have moved up or down, but simply to have kept to this one plane. Suppose, that is, some figure, such as a circle or rectangle, to be endowed with the power of perception; such a being if it moves in the plane superficies in which it is drawn, will move in a multitude of directions; but, however varied they may seem to be, these directions will all be compounded of two, at right angles to each other. By no movement so long as the plane superficies remains perfectly horizontal, will this being move in the direction we call up and down. And it is important to notice that the plane would be different to a creature confined to it, from what it is to us. We think of a plane habitually as having an upper and a lower side, because it is only by the contact of solids that we realize a plane. But a creature which had been confined to a plane during its whole existence would have no idea of there being two sides to the plane he lived in. In a

[1] This question still occupies many philosophers of science. A comprehensive discussion of the work which has been done appears in Max Jammer's book, *The Concept of Space*. [R.v.B.R.]

plane there is simply length and breadth. If a creature in it be supposed to know of an up or down he must already have gone out of the plane.

Is it possible, then, that a creature so circumstanced would arrive at the notion of there being an up and down, a direction different from those to which he had been accustomed, and having nothing in common with them? Obviously nothing in the creature's circumstances would tell him of it. It could only be by a process of reasoning on his part that he could arrive at such a conception. If he were to imagine a being confined to a single straight line, he might realize that he himself could move in two directions, while the creature in a straight line could only move in one. Having made this reflection he might ask, "But why is the number of directions limited to two? Why should there not be three?"

A creature (if such existed), which moves in a plane would be much more fortunately circumstanced than one which can only move in a straight line. For, in a plane, there is a possibility of an infinite variety of shapes, and the being we have supposed could come into contact with an indefinite number of other beings. He would not be limited, as in the case of the creature in a straight line, to one only on each side of him.

It is obvious that it would be possible to play curious tricks with a being confined to a plane. If, for instance, we suppose such a being to be inside a square, the only way out that he could conceive would be through one of the sides of the square. If the sides were impenetrable, he would be a fast prisoner, and would have no way out.

What his case would be we may understand, if we reflect what a similar case would be in our own existence. The creature is shut in in all the directions he knows of. If a man is shut in in all the directions he knows of, he must be surrounded by four walls, a roof and a floor. A two-dimensional being inside a square would be exactly in the same predicament that a man would be, if he were in a room with no opening on any side. Now it would be possible to us to take up such a being from the inside of the square, and to set him down outside it. A being to whom this had happened would find himself outside the place he had been confined in, and he would not have passed through any of the boundaries by which he was shut in. The astonishment of such a being can only be imagined by comparing it to that which a man would feel, if he were suddenly to find himself outside a room in which

he had been, without having passed through the window, doors, chimney or any opening in the walls, ceiling or floor.

Another curious thing that could be effected with a two-dimensional being, is the following. Conceive two beings at a great distance from one another on a plane surface. If the plane surface is bent so that they are brought close to one another, they would have no conception of their proximity, because to each the only possible movements would seem to be movements in the surface. The two beings might be conceived as so placed, by a proper bending of the plane, that they should be absolutely in juxtaposition, and yet to all the reasoning faculties of either of them a great distance could be proved to intervene. The bending might be carried so far as to make one being suddenly appear in the plane by the side of the other. If these beings were ignorant of the existence of a third dimension, this result would be as marvellous to them, as it would be for a human being who was at a great distance—it might be at the other side of the world—to suddenly appear and really be by our side, and during the whole time he not to have left the place in which he was.

Constructing a Four-Square

The foregoing examples make it clear that beings can be conceived as living in a more limited space than ours. Is there a similar limitation in the space we know?

At the very threshold of arithmetic an indication of such a limitation meets us.

If there is a straight line before us two inches long, its length is expressed by the number 2. Suppose a square to be described on the line, the number of square inches in this figure is expressed by the number 4, i.e., 2×2. This 2×2 is generally written 2^2, and named "2 squared."

Now, of course, the arithmetical process of multiplication is in no sense identical with that process by which a square is generated from the motion of a straight line, or a cube from the motion of a square. But it has been observed that the units resulting in each case, though different in kind, are the same in number.

If we touch two things twice over, the act of touching has been performed four times. Arithmetically, $2 \times 2 = 4$. If a square is generated by the motion of a line two inches in length, this square contains four square inches.

So it has come to pass that the second and third powers of numbers are called "square" and "cube."

We have now a straight line two inches long. On this a square has been constructed containing four square inches. If on the same line a cube be constructed, the number of cubic inches in the figure so made is 8, i.e., $2 \times 2 \times 2$ or 2^3. Here, corresponding to the numbers 2, 2^2, 2^3, we have a series of figures. Each figure contains more units than the last, and in each the unit is of a different kind. In the first figure a straight line is the unit, viz., one linear inch; it is said to be of one dimension. In the second a square is the unit, viz., one square inch. The square is a figure of two dimensions. In the third case a cube is the unit, and the cube is of three dimensions. The straight line is said to be of one dimension because it can be measured only in one way. Its length can be taken, but it has no breadth or thickness. The square is said to be of two dimensions because it has both length and breadth. The cube is said to have three dimensions, because it can be measured in three ways.

The question naturally occurs, looking at these numbers 2, 2^2, 2^3, by what figure shall we represent 2^4, or $2 \times 2 \times 2 \times 2$. We know that in the figure there must be sixteen units, or twice as many units as in the cube. But the unit also itself must be different. And it must not differ from a cube simply in shape. It must differ from a cube as a cube differs from a square. No number of squares will make up a cube, because each square has no thickness. In the same way, no number of cubes must be able to make up this new unit. And here, instead of trying to find something already known, to which the idea of a figure corresponding to the fourth power can be affixed, let us simply reason out what the properties of such a figure must be. In this attempt we have to rely, not on a process of touching or vision, such as informs us of the properties of bodies in the space we know, but on a process of thought. Each fact concerning this unknown figure has to be reasoned out; and it is only after a number of steps have been gone through, that any consistent familiarity with its properties is obtained. Of all applications of the reason, this exploration is perhaps the one which requires, for the simplicity of the data involved, the greatest exercise of the abstract imagination, and on this account is well worth patient attention. The first steps are very simple. We must imagine a finite straight line to generate a square by moving on the plane of the paper, and this square in its turn to generate a cube by moving vertically upwards. Figure 1 represents a straight line; figure 2 represents a

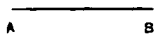


FIGURE 1

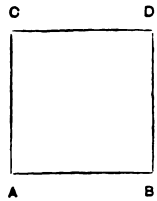


FIGURE 2

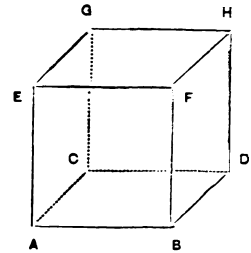


FIGURE 3

square formed by the motion of that straight line; figure 3 represents perspectively a cube formed by the motion of that square *ABCD* upwards. It would be well, instead of using figure 3, to place a cube on the paper. Its base would be *ABCD*, its upper surface *EFGH*.

The straight line *AB* gives rise to the square *ABCD* by a movement at right angles to itself. If motion be confined to the straight line *AB*, a backward and forward motion is the only one possible. No sideway motion is admissible. And if we suppose a being to exist which could only move in the straight line *AB*, it would have no idea of any other movement than to and fro. The square *ABCD* is formed from the straight line by a movement in a direction entirely different from the direction which exists in *AB*. This motion is not expressible by means of any possible motion in *AB*. A being which existed in *AB*, and whose experience was limited to what could occur in *AB*, would not be able to understand the instructions we should give to make *AB* trace out the figure *ABCD*.

In the figure *ABCD* there is a possibility of moving in a variety of directions, so long as all these directions are confined to one plane. All directions in this plane can be considered as compounded of two, from *A* to *B*, and from *A* to *C*. Out of the infinite variety of such directions there is none which tends in a direction perpendicular to figure 2; there is none which tends upwards from the plane of the paper. Conceive a being to exist in the plane, and to move only in it. In all the movements which he went through there would be none by which he could conceive the alteration of figure 2 into what figure 3 represents in perspective. For 2 to become 3 it must be supposed to move perpendicularly to its own plane. The figure it traces out is the cube *ABCDEFGH*.

All the directions, manifold as they are, in which a creature existing in figure 3 could move, are compounded of three directions. From *A* to

B, from A to C, from A to E, and there are no other directions known to it.

But if we suppose something similar to be done to figure 3, something of the same kind as was done to figure 1 to turn it into figure 2, or to figure 2 to turn it into figure 3, we must suppose the whole figure as it exists to be moved in some direction entirely different from any direction within it, and not made up of any combination of the directions in it. What is this? It is the fourth direction.

We are as unable to imagine it as a creature living in the plane figure 2 would be to imagine a direction such that moving in it the square 2 would become the cube 3. The third dimension to such a creature would be as unintelligible as the fourth is to us. And at this point we have to give up the aid that is to be got from any presentable object, and we have simply to investigate what the properties of the simplest figure in four dimensions are, by pursuing further the analogy which we know to exist between the process of formation of 2 from 1, and of 3 from 2, and finally of 4 from 3. For the sake of convenience, let us call the figure we are investigating—the simplest figure in four dimensions—a *four-square*.

First of all we must notice, that if a cube be formed from a square by the movement of the square in a new direction, each point of the interior of the square traces out part of the cube. It is not only the bounding lines that by their motion form the cube, but each portion of the interior of the square generates a portion of the cube. So if a cube were to move in the fourth dimension so as to generate a four-square, every point in the interior of the cube would start *de novo*, and trace out a portion of the new figure uninterfered with by the other points.

Or, to look at the matter in another light, a being in three dimensions, looking down on a square, sees each part of it extended before him, and can touch each part without having to pass through the surrounding parts, for he can go from above, while the surrounding parts surround the part he touches only in one plane.

So a being in four dimensions could look at and touch every point of a solid figure. No one part would hide another, for he would look at each part from a direction which is perfectly different from any in which it is possible to pass from one part of the body to another. To pass from one part of the body to another it is necessary to move in three directions, but a creature in four dimensions would look at the solid from a direction which is none of these three.

Let us obtain a few facts about the fourth figure, proceeding according to the analogy that exists between 1, 2, 3, and 4. In the figure 1 there are two points. In 2 there are four points—the four corners of the square. In 3 there are eight points. In the next figure, proceeding according to the same law, there would be sixteen points.

In the figure 1 there is one line. In the square there are four lines. In the cube there are twelve lines. How many lines would there be in the four-square? That is to say that there are three numbers—1, 4, and 12. What is the fourth, going on accordingly to the same law?

To answer this question let us trace out in more detail how the figures change into one another. The line, to become the square, moves; it occupies first of all its original position, and last of all its final position. It starts as AB, and ends as CD; thus the line appears twice, or it is doubled. The two other lines in the square, AC, BD, are formed by the motions of the points at the extremities of the moving line. Thus, in passing from the straight line to the square the lines double themselves, and each point traces out a line. If the same procedure holds good in the case of the change of the square into the cube, we ought in the cube to have double the number of lines as in the square—that is eight—and every point in the square ought to become a line. As there are four points in the square, we should have four lines in the cube from them, that is, adding to the previous eight, there should be twelve lines in the cube. This is obviously the case. Hence we may with confidence, to deduce the number of lines in a four-square, apply this rule. *Double the number of lines in the previous figure, and add as many lines as there are points in the previous figure.* Now in the cube there are twelve lines and eight points. Hence we get $2 \times 12 + 8$, or thirty-two lines in the four-square.

In the same way any other question about the four-square can be answered. We must throw aside our realizing power and answer in accordance with the analogy to be worked out from the three figures we know.

Thus, if we want to know how many plane surfaces the four-square has, we must commence with the line, which has none; the square has one; the cube has six. Here we get the three numbers, 0, 1, and 6. What is the fourth?

Consider how the planes of the cube arise. The square at the beginning of its motion determines one of the faces of the cube, at the end it is the opposite face, during the motion each of the lines of the square

traces out one plane face of the cube. Thus we double the number of planes in the previous figure, and every line in the previous figure traces out a plane in the subsequent one.

Apply this rule to the formation of a square from a line. In the line there is no plane surface, and since twice nothing is nothing, we get, so far, no surface in the square; but in the straight line there is one line, namely itself, and this by its motion traces out the plane surface of the square. So in the square, as should be, the rule gives one surface.

Applying this rule to the case of the cube, we get, doubling the surfaces, 12; and adding a plane for each of the straight lines, of which there are 12, we have another 12, or 24 plane surfaces in all. Thus, just as by handling or looking at it, it is possible to describe a figure in space, so by going through a process of calculation it is within our power to describe all the properties of a figure in four dimensions.

There is another characteristic so remarkable as to need a special statement. In the case of a finite straight line, the boundaries are points. If we deal with one dimension only, the figure 1, that of a segment of a straight line, is cut out of and separated from the rest of an imaginary infinitely long straight line by the two points at its extremities. In this simple case the two points correspond to the bounding surface of the cube. In the case of a two-dimensional figure an infinite plane represents the whole of space. The square is separated off by four straight lines, and it is impossible for an entry to be made into the interior of the square, except by passing through the straight lines. Now, in these cases, it is evident that the boundaries of the figure are of one dimension less than the figure itself. Points bound lines, lines bound plane figures, planes bound solid figures. Solids then must bound four dimensional figures. The four-square will be bounded in the following manner. First of all there is the cube which, by its motion in the fourth direction, generates the figure. This, in its initial position, forms the base of the four-square. In its final position it forms the opposite end. During the motion each of the faces of the cube give rise to another cube. The direction in which the cube moves is such that of all the six sides none is in the least inclined in that direction. It is at right angles to all of them. The base of the cube, the top of the cube, and the four sides of the cube, each and all of them form cubes. Thus the four-square is bounded by eight cubes. Summing up, the four-square would have 16 points, 32 lines, 24 surfaces, and it would be bounded by 8 cubes.

If a four-square were to rest in space it would seem to us like a cube.

To justify this conclusion we have but to think of how a cube would appear to a two-dimensional being. To come within the scope of his faculties at all, it must come into contact with the plane in which he moves. If it is brought into as close a contact with this plane as possible, it rests on it by one of its faces. This face is a square, and the most a two-dimensional being could get acquainted with of a cube would be a square.

Having thus seen how it is possible to describe the properties of the simplest shape in four dimensions, it is evident that the mental construction of more elaborate figures is simply a matter of time and patience.

In the study of the form and development of the chick in the egg, it is impossible to detect the features that are sought to be observed, except by the use of the microscope. The specimens are accordingly hardened by a peculiar treatment and cut into thin sections. The investigator going over each of these sections, noticing all their peculiarities, constructs in his mind the shape as it originally existed from the record afforded by an indefinite number of slices. So, to form an idea of a four-dimensional figure, a series of solid shapes bounded on every side differing gradually from one another, proceeding, it may be, to the most diverse forms, has to be mentally grasped and fused into a unitary conception.

If, for instance, a small sphere were to appear, this to be replaced by a larger one, and so on, and then, when the largest had appeared, smaller and smaller ones to make their appearance, what would be witnessed would be a series of sections of a four-dimensional sphere. Each section in space being a sphere.

Again, just as solid figures can be represented on paper by perspective, four-dimensional figures can be represented perspectively by solids. If there are two squares, one lying over the other, and the underneath one be pushed away, its sides remaining parallel with the one that was over it, then if each point of the one be joined to the corresponding point of the other, we have a fair representation on paper of a cube. Figure 3 may be considered to be such a representation if the square *CDGH* be considered to be the one that has been pushed away from lying originally under the square *ABEF*. Each of the planes which bound the cube is represented on the paper. The only thing that is wanting is the three-dimensional content of the cube. So if

two cubes be placed with their sides parallel, but one somewhat diagonally with regard to the other, and all their corresponding points be supposed joined, there will be found a set of solid figures, each representing (though of course distortedly) the bounding cubes of the four-dimensional figure, and every plane and line in the four-dimensional figure will be found to be represented in a kind of solid perspective. What is wanting is of course the four-dimensional content.

Properties of Matter in the Fourth Dimension

Having now passed in review some of the properties of four-dimensional figures, it remains to ask what relations beings in four dimensions, if they did exist, would have with us.

And in the first place, a being in four dimensions would have to us exactly the appearance of a being in space. A being in a plane would only know solid objects as two-dimensional figures—the shapes namely in which they intersected his plane. So if there were four-dimensional objects, we should only know them as solids—the solids, namely, in which they intersect our space. Why, then, should not the four-dimensional beings be ourselves, and our successive states the passing of them through the three-dimensional space to which our consciousness is confined?

Let us consider the question in more detail. And for the sake of simplicity transfer the problem to the case of three and two dimensions instead of four and three.

Suppose a thread to be passed through a thin sheet of wax placed horizontally. It can be passed through in two ways. Either it can be pulled through, or it can be held at both ends, and moved downwards as a whole. Suppose a thread to be grasped at both ends, and the hands to be moved downwards perpendicularly to the sheet of wax. If the thread happens to be perpendicular to the sheet it simply passes through it, but if the thread be held, stretched slantingwise to the sheet, and the hands are moved perpendicularly downwards, the thread will, if it be strong enough, make a slit in the sheet.

If now the sheet of wax were to have the faculty of closing up behind the thread, what would appear in the sheet would be a moving hole.

Suppose that instead of a sheet and a thread, there were a straight line and a plane. If the straight line were placed slantingwise in reference to the plane and moved downwards, it would always cut the

plane in a point, but that point of section would move on. If the plane were of such a nature as to close up behind the line, if it were of the nature of a fluid, what would be observed would be a moving point. If now there were a whole system of lines sloping in different directions, but all connected together, and held absolutely still by one framework, and if this framework with its system of lines were as a whole to pass slowly through the fluid plane at right angles to it, there would then be the appearance of a multitude of moving points in the plane, equal in number to the number of straight lines in the system. The lines in the framework will all be moving at the same rate—namely, at the rate of the framework in which they are fixed. But the points in the plane will have different velocities. They will move slower or faster, according as the lines which give rise to them are more or less inclined to the plane. A straight line perpendicular to the plane will, on passing through, give rise to a stationary point. A straight line that slopes very much inclined to the plane will give rise to a point moving with great swiftness.[2] The motions and paths of the points would be determined by the arrangement of the lines in the system. It is obvious that if two straight lines were placed lying across one another like the letter X, and if this figure were to be stood upright and passed through the plane, what would appear would be at first two points. These two points would approach one another. When the part where the two strokes of the X meet came into the plane, the two points would become one. As the upper part of the figure passed through, the two points would recede from one another.

If the line be supposed to be affixed to all parts of the framework, and to loop over one another, and support one another (*ABCD* framework, *X* and *Y* two lines interlinked), it is obvious that they could assume all sorts of figures, and that the points on the plane would move in very complicated paths. Figure 4 represents a section of such a framework. Two lines *XX* and *YY* are shown, but there must be supposed to be a great number of others sloping backwards and forwards as well as sideways.

Let us now assume that instead of lines, very thin threads were attached to the framework: they on passing through the fluid plane would give rise to very small spots. Let us call the spots atoms, and

[2] Hinton is here anticipating the Minkowskian notion of using a higher dimension to get a static representation of spacetime. He was, of course, unaware of the relativistic restriction which forbids world-lines with an inclination so steep that they would represent particles moving faster than the speed of light. [R.v.B.R.]

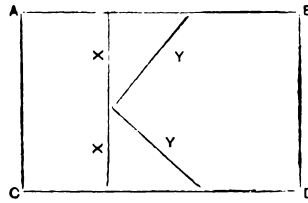


FIGURE 4

regard them as constituting a material system in the plane. There are four conditions which must be satisfied by these spots if they are to be admitted as forming a material system such as ours. For the ultimate properties of matter (if we eliminate attractive and repulsive forces, which may be caused by the motions of the smallest particles), are—1, Permanence; 2, Impenetrability; 3, Inertia; 4, Conservation of energy.

According to the first condition, or that of permanence, no one of these spots must suddenly cease to exist. That is, the thread which by sharing in the general motion of the system gives rise to the moving point, must not break off before the rest of them. If all the lines suddenly ended this would correspond to a ceasing of matter.

2. Impenetrability.—One spot must not pass through another. This condition is obviously satisfied. If the threads do not coincide at any point, the moving spots they give rise to cannot.

3. Inertia.—A spot must not cease to move or cease to remain at rest without coming into collision with another point. This condition gives the obvious condition with regard to the threads, that they, between the points where they come into contact with one another, must be straight. A thread which was curved would, passing through the plane, give rise to a point which altered in velocity spontaneously. This the particles of matter never do.

4. Conservation of energy.—The energy of a material system is never lost; it is only transferred from one form to another, however it may seem to cease. If we suppose each of the moving spots on the plane to be the unit of mass, the principle of the conservation of energy demands that when any two meet, the sum of the squares of their several velocities before meeting shall be the same as the sum of the squares of their velocities after meeting. Now we have seen that any statement about the velocities of the spots in the plane is really a statement about the inclinations of the threads to the plane. Thus the principle of the conservation of energy gives a condition which must be

satisfied by the inclinations of the threads of the plane. Translating this statement, we get in mathematical language the assertion that the sum of the squares of the tangents of the angles the threads make with the normal to the plane remains constant.

Hence, all complexities and changes of a material system made up of similar atoms in a plane could result from the uniform motion as a whole of a system of threads.

We can imagine these threads as weaving together to form connected shapes, each complete in itself, and these shapes as they pass through the fluid plane give rise to a series of moving points. Yet, inasmuch as the threads are supposed to form consistent shapes, the motion of the points would not be wholly random, but numbers of them would present the semblance of moving figures. Suppose, for instance, a number of threads to be so grouped as to form a cylinder for some distance, but after a while to be pulled apart by other threads with which they interlink. While the cylinder was passing through the plane, we should have in the plane a number of points in a circle. When the part where the threads deviated came to the plane, the circle would break up by the points moving away. These moving figures in the plane are but the traces of the shapes of threads as those shapes pass on. These moving figures may be conceived to have a life and a consciousness of their own.

Or, if it be irrational to suppose them to have a consciousness when the shapes of which they are momentary traces have none, we may well suppose that the shapes of threads have consciousness, and that the moving figures share this consciousness, only that in their case it is limited to those parts of the shapes that simultaneously pass through the plane. In the plane, then, we may conceive bodies with all the properties of a material system, moving and changing, possessing consciousness. After a while it may well be that one of them becomes so disassociated that it appears no longer as a unit, and its consciousness as such may be lost. But the threads of existence of such a figure are not broken, nor is the shape which gave it origin altered in any way. It has simply passed on to a distance from the plane. Thus nothing which existed in the conscious life on the plane would cease. There would in such an existence be no cause and effect, but simply the gradual realization in a superficies of an already existent whole. There would be no progress, unless we were to suppose the threads as they pass to interweave themselves in more complex shapes.

Can a representation, such as the preceding, be applied to the case of the existence in space with which we have to do? Is it possible to suppose that the movements and changes of material objects are the intersections with a three-dimensional space of a four-dimensional existence? Can our consciousness be supposed to deal with a spatial profile of some higher actuality?

It is needless to say that all the considerations that have been brought forward in regard to the possibility of the production of a system satisfying the conditions of materiality by the passing of threads through a fluid plane, holds good with regard to a four-dimensional existence passing through a three-dimensional space. Each part of the ampler existence which passed through our space would seem perfectly limited to us. We should have no indication of the permanence of its existence. Were such a thought adopted, we should have to imagine some stupendous whole, wherein all that has ever come into being or will come co-exists, which passing slowly on, leaves in this flickering consciousness of ours, limited to a narrow space and a single moment, a tumultuous record of changes and vicissitudes that are but to us. Change and movement seem as if they were all that existed. But the appearance of them would be due merely to the momentary passing through our consciousness of ever existing realities.

In thinking of these matters it is hard to divest ourselves of the habit of visual or tangible illustration. If we think of a man as existing in four dimensions, it is hard to prevent ourselves from conceiving him prolonged in an already known dimension. The image we form resembles somewhat those solemn Egyptian statues which in front represent well enough some dignified sitting figure, but which are immersed to their ears in a smooth mass of stone which fits their contour exactly.

No material image will serve. Organized beings seem to us so complete that any addition to them would deface their beauty. Yet were we creatures confined to a plane, the outline of a Corinthian column would probably seem to be of a beauty unimprovable in its kind. We should be unable to conceive any addition to it, simply for the reason that any addition we could conceive would be of the nature of affixing an unsightly extension to some part of the contour. Yet, moving as we do in space of three dimensions, we see that the beauty of the stately column far surpasses that of any single outline. So all that we can do is to deny our faculty of judging of the ideal completeness of shapes in four dimensions.

Let us now leave this supposition of framework and threads. Let us investigate the conception of a four-dimensional existence in a simpler and more natural manner—in the same way that a two-dimensional being should think about us, not as infinite in the third dimension, but limited in three dimensions as he is in two.[3] A being existing in four dimensions must then be thought to be as completely bounded in all four directions as we are in three. All that we can say in regard to the possibility of such beings is, that we have no experience of motion in four directions. The powers of such beings and their experience would be ampler, but there would be no fundamental difference in the laws of force and motion.

Such a being would be able to make but a part of himself visible to us, for a cube would be apprehended by a two-dimensional being as the square in which it stood. Thus a four-dimensional being would suddenly appear as a complete and finite body, and as suddenly disappear, leaving no trace of himself, in space, in the same way that anything lying on a flat surface, would, on being lifted, suddenly vanish out of the cognizance of beings, whose consciousness was confined to the plane. The object would not vanish by moving in any direction, but disappear instantly as a whole. There would be no barrier, no confinement of our devising that would not be perfectly open to him. He would come and go at pleasure; he would be able to perform feats of the most surprising kind. It would be possible by an infinite plane extending in all directions to divide our space into two portions absolutely separated from one another; but a four-dimensional being would slip round this plane with the greatest ease.

To see this clearly, let us first take the analogous case in three dimensions. Suppose a piece of paper to represent a plane. If it is infinitely extended in every direction, it will represent an infinite plane. It can be divided into two parts by an infinite straight line. A being confined to this plane could not get from one part of it to the other without passing through the line. But suppose another piece of paper laid on the first and extended infinitely, it will represent another infinite plane. If the being moves from the first plane by a motion in the third dimension, it will move into this new plane. And in it it finds no

[3] Here Hinton shifts the attention from 4-D spacetime to the quite different notion of pure 4-D space. [R.v.B.R.]

line. Let it move to such a position that when it goes back to the first plane it will be on the other side of the line. Then let it go back to the first plane. It has appeared now on the other side of the line which divides the infinite plane into two parts.

Take now the case of four dimensions. Instead of bringing before the mind a sheet of paper conceive a solid of three dimensions. If this solid were to become infinite it would fill up the whole of three-dimensional space. But it would not fill up the whole of four-dimensional space. It would be to four-dimensional space what an infinite plane is to three-dimensional space. There could be in four-dimensional space an infinite number of such solids, just as in three-dimensional space there could be an infinite number of infinite planes.

Thus, lying alongside our space, there can be conceived a space also infinite in all three directions. To pass from one to the other a movement has to be made in the fourth dimension, just as to pass from one infinite plane to another a motion has to be made in the third dimension.

Conceive, then, corresponding to the first sheet of paper mentioned above, a solid, and as the sheet of paper was supposed to be infinitely extended in two dimensions, suppose the solid to be infinitely extended in its three dimensions, so that it fills the whole of space as we know it.

Now divide this infinite solid in two parts by an infinite plane, as the infinite plane of paper was divided in two parts by an infinite line. A being cannot pass from one part of this infinite solid to another, on the other side of this infinite plane, without going through the infinite plane, *so long as he keeps within the infinite solid.*

But suppose beside this infinite solid a second infinite solid, lying next to it in the fourth dimension, as the second infinite plane of paper was next to the first infinite plane in the third dimension. Let now the being that wants to get on the other side of the dividing plane move off in the fourth dimension, and enter the second infinite solid. In this second solid there is no dividing plane. Let him now move, so that coming back to the first infinite solid he shall be on the other side of the infinite plane that divides it into two portions. If this is done, he will now be on the other side of the infinite plane, without having gone through it.

In a similar way a being, able to move in four dimensions, could get out of a closed box without going through the sides, for he could move

off in the fourth dimension, and then move about, so that when he came back he would be outside the box.

Is there anything in the world as we know it, which would indicate the possibility of there being an existence in four dimensions? No definite answer can be returned to this question. But it may be of some interest to point out that there are certain facts which might be read by the light of the fourth-dimensional theory.

To make this clear, let us suppose that space is really four dimensional, and that the three-dimensional space we know is, in this ampler space, like a surface is in our space.

We should then be in this ampler space like beings confined to the surface of a plane would be in ours. Let us suppose that just as in our space there are centers of attraction whose influence radiates out in every direction, so in this ampler space there are centers of attraction whose influence radiates out in every direction. Is there anything to be observed in nature which would correspond to the effect of a center of attraction lying out of our space, and acting on all the matter in it? The effect of such a center of attraction would not be to produce motion in any known direction, because it does not lie off in any known direction.

Let us pass to the corresponding case in three and two dimensions, instead of four and three. Let us imagine a plane lying horizontally, and in it some creatures whose experience was confined to it. If now some water or other liquid were poured on to the plane, the creatures, becoming aware of its presence, would find that it had a tendency to spread out all over the plane. In fact it would not be to them as a liquid is to us—it would rather correspond to a gas. For a gas, as we know it, tends to expand in every direction, and gradually increase so as to fill the whole of space. It exercises a pressure on the walls of any vessel in which we confine it.

The liquid on the plane expands in all the dimensions which the two-dimensional creatures on the plane know, and at the same time becomes smaller in the third dimension, its absolute quantity remaining unchanged. In like manner we might suppose that gases (which by expansion become larger in the dimensions that we know) become smaller in the fourth dimension.

The cause in this case would have to be sought for in an attractive force, acting with regard to our space as the force of gravity acts with regard to a horizontal plane.

Can we suppose that there is a center of attraction somewhere off in the fourth dimension, and that the gases, which we know are simply more mobile liquids, expanding out in every direction under its influence. This view receives a certain amount of support from the fact proved experimentally that there is no absolute line of demarcation between a liquid and a gas. The one can be made to pass into the other with no moment intervening in which it can be said that now a change of state has taken place.

We might then suppose that the matter we know extending in three dimensions has also a small thickness in the fourth dimension; that solids are rigid in the fourth as in the other three dimensions; that liquids are too coherent to admit of their spreading out in space, and becoming thinner in the fourth dimension, under the influence of an attractive center lying outside of our space; but that gases, owing to the greater mobility of their particles, are subject to its action, and spread out in space under its influence, in the same manner that liquids, under the influence of gravity, spread out on a plane.

Then the density of a gas would be a measure of the relative thickness of it in the fourth dimension: and the diminution of the density would correspond to a diminution of the thickness in the fourth dimension. Could this supposition be tested in any way?

Suppose a being confined to a plane; if the plane is moved far off from the center of attraction lying outside it, he would find that liquids had less tendency to spread out than before.

Or suppose he moves to a distant part of the plane so that the line from his position to the center of attraction lies obliquely to the plane; he would find that in this position a liquid would show a tendency to spread out more in one direction than another.

Now our space considered as lying in four-dimensional space, as a plane does in three-dimensional space, may be shifted. And the expansive force of gases might be found to be different at different ages. Or, shifting as we do our position in space during the course of the earth's path round the sun, there might arise a sufficient difference in our position in space, with regard to the attractive center, to make the expansive force of gases different at different times of the year, or to cause them to manifest a greater expansive force in one direction than in another.[4]

But although this supposition might be worked out at some length, it is hard to suppose that it could afford any definite test of the physical existence of a fourth dimension. No test has been discovered which is decisive. And, indeed, before searching for tests, a theoretical point of the utmost importance has to be settled. In discussing the geometrical properties of straight lines and planes, we suppose them to be respectively of one and two dimensions, and by so doing deny them any real existence. A plane and a line are mere abstractions. Every portion of matter is of three dimensions. If we consider beings on a plane not as mere idealities, we must suppose them to be of some thickness.[5] If their experience is to be limited to a plane this thickness must be very small compared to their other dimensions. Transferring our reasoning to the case of four dimensions, we come to a curious result.

If a fourth dimension exists there are two possible alternatives.

One is, that there being four dimensions, we have a three-dimensional existence only. The other is that we really have a four-dimensional existence, but are not conscious of it. If we are in three dimensions only, while there are really four dimensions, then we must be relatively to those beings who exist in four dimensions, as lines and planes are in relation to us. That is, we must be mere abstractions. In this case we must exist only in the mind of the being that conceives us, and our experience must be merely the thoughts of his mind—a result which has apparently been arrived at, on independent grounds, by an idealist philosopher.

The other alternative is that we have a four-dimensional existence. In this case our proportions in it must be infinitely minute, or we should be conscious of them. If such be the case, it would probably be in the ultimate particles of matter, that we should discover the fourth dimension, for in the ultimate particles the sizes in the three dimensions are very minute, and the magnitudes in all four dimensions would be comparable.

The preceding two alternative suppositions are based on the hypothesis of the reality of four-dimensional existence, and must be conceived to hold good only on that hypothesis.

[5] To see how one can have an ideal plane world of no thickness, just choose any arbitrary plane, say the top of your dining table, and consider the world formed by taking the intersections of the objects in our world with this plane. Note that the moving 2-D shapes in this world would obey certain laws—e.g., a large, roughly circular blob would hover near the table three times a day. [R.v.B.R.]

It is somewhat curious to notice that we can thus conceive of an existence relative to which that which we enjoy must exist as a mere abstraction.

Apart from the interest of speculations of this kind they have considerable value; for they enable us to express in intelligible terms things of which we can form no image. They supply us, as it were, with scaffolding, which the mind can make use of in building up its conceptions. And the additional gain to our power of representation is very great.

Many philosophical ideas and doctrines are almost unintelligible because there is no physical illustration which will serve to express them. In the imaginary physical existence which we have traced out, much that philosophers have written finds adequate representation. Much of Spinoza's Ethics, for example, could be symbolized from the preceding pages.

Thus we may discuss and draw perfectly legitimate conclusions with regard to unimaginable things.

It is, of course, evident that these speculations present no point of direct contact with fact. But this is no reason why they should be abandoned. The course of knowledge is like the flow of some mighty river, which, passing through the rich lowlands, gathers into itself the contributions from every valley. Such a river may well be joined by a mountain stream, which, passing with difficulty along the barren highlands, flings itself into the greater river down some precipitous descent, exhibiting at the moment of its union the spectacle of the utmost beauty of which the river system is capable. And such a stream is no inapt symbol of a line of mathematical thought, which, passing through difficult and abstract regions, sacrifices for the sake of its crystalline clearness the richness that comes to the more concrete studies. Such a course may end fruitlessly, for it may never join the main course of observation and experiment. But, if it gains its way to the great stream of knowledge, it affords at the moment of its union the spectacle of the greatest intellectual beauty, and adds somewhat of force and mysterious capability to the onward current.

A PLANE WORLD

[This selection is reprinted in its entirety from *Scientific Romances,* Vol. 1 (1884).]

Where the sun's rays grazing the earth in January pass off and merge into darkness lies a strange world.

'Tis a vast bubble blown in a substance something like glass, but harder far and untransparent.

And just as a bubble blown by us consists of a distended film, so this bubble, vast beyond comparison, consists of a film distended and coherent.

On its surface in the course of ages has fallen a thin layer of space dust, and so smooth is this surface that the dust slips over it to and fro and forms densities and clusters as its own attractions and movements determine.

The dust is kept on the polished surface by the attraction of the vast film; but, except for that, it moves on it freely in every direction.

And here and there are condensations wherein have fallen together numbers of these floating masses, and where the dust condensing for ages has formed vast disks.

And these disks are glowing hot—yet no light comes from them into our universe.

For this world lies beyond the aether—far beyond. And however hot or glowing the masses are, if there is no medium to transmit the vibrations of heat the influence cannot travel.

Thus the only directions in which the heat can travel are on the film. From each of these glowing disks the luminous influence streams forth carried by the vibrations of the film which supports everything. For the heat and intense agitation of these glowing disks shakes and disturbs the bubble, and just as a thin soap bubble quivers and shakes, so this film quivers and shakes. And so elastic is it, and so rigid, that it carries the light and heat to all surrounding regions. Yet so vast is the bubble, so tremendous in its dimensions, that the agitation from these glowing disks travels almost in straight lines, till, spreading out on

every side, it merges into darkness—like the ripples in the center of a vast calm lake gradually become indistinguishable.

And round these central orbs of fire—for orbs of fire they are, though they only transmit their fire along the film of the bubble—round these orbs pass in due order and succession other disks, which, cold or warm, have not that energy of light and heat which the central orbs possess.

These disks, though large, are so immeasurably small compared with the vast surface of the all-supporting bubble, that their movements seem to lie on a plane flat surface; the curving of the film on which they rest is so slight compared to their magnitude, that they sail round and round their central fires as on a perfect level surface.

And one of these orbs is fitted by nature to be the habitation and home of living beings. For it is neither so hot as it was for long ages after it had condensed from the film of dust wherefrom all orbs are made, nor has it so cooled down as to render life unsupportable.

And, moreover, it is full of vast crevices and channels, for in many places the interior in cooling after the rim had set from its molten condition has left long caverns and passages, not only in one layer, but in many.

And on the rim and in these passages and caverns live the inhabitants of whom I speak.

They do not rise from the surface of the film, but as all matter lies on the smooth surface but one particle deep, so their bodies formed out of matter lie, as we should say, on this smooth surface.

Yet of this they know nothing. They say that they stand and walk.

For this orb has an attractive force.

By that very same impulse of coming together whereby it gathered its particles out of the dust on the bubble, by that very same force it draws towards its center all that is near it or on it. Thus "up" is to

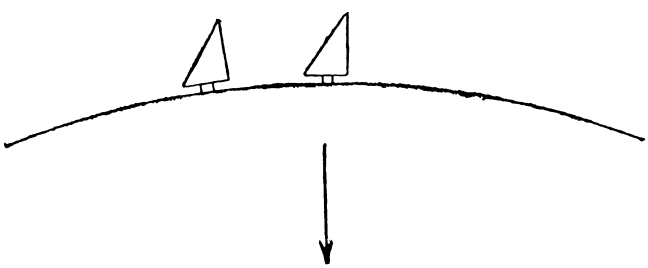


FIGURE 5. Two beings walking round.

these inhabitants a movement from the center of the disk on the rim of which they live and away from it. "Down" is a movement from the rim towards the center. The thin layer which forms the mass of the disk is their solid matter. They are not able even in thought to rise away from the surface of the bubble, and look from space upon their mode of existence. They ever pass to and fro upon a line, upon a rim; and no two can walk except after one another. If you look at the rude picture you will see that the two beings represented by two triangles cannot pass one another if they are unable to lift themselves up from lying on the surface of the paper. The surface of the paper represents the surface of the bubble, and sliding freely on it, but unable to lift up from it, are tenuous shapes that are the inhabitants, and that thin layer of particles that is to them solid matter.

Now were it not for the fact that the orb is reft into these chasms and passages, the only movement that these beings would have would be of passing round and round on the rim of their world.

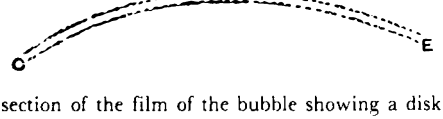


FIGURE 6. A section of the film of the bubble showing a disk BD lying on it, and a creature AB on the rim of the disk. CE is a section of the film, BD is a section of the disk, AB is a section of the creature. The thickness is enormously magnified and also the height AB of the creature compared with the diameter BD of the disk. The attraction which AB feels keeps him to BD; both AB and BD, the being and the disk, slide freely on the film CE without knowing of its existence.

Many words that we have, to them could bear no meaning. Thus "right and left" is to them unknown. For consider their faces bent in one direction along the rim. In following this direction, they go forward, in retracting from it they go backward. If they go away from the center they go up, towards the center is down. And by no means can they turn, raising themselves from the surface whereon they are. They do not even know that they have two sides; their movements, thoughts, and imaginations are all confined to that surface on which they are. This they call their space, their universe; nor does aught that lies beyond it, towards the interior of the bubble or away from it, directed outwards, come into their thoughts, even as an imaginary possibility of existence.

Life is extremely limited on such a world. To take a single instance,

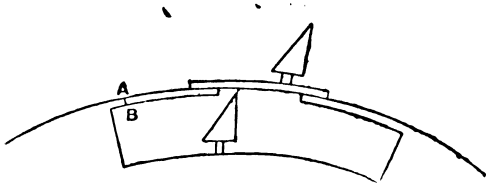


FIGURE 7. Two beings passing.

in order for two beings to pass each other, a complicated arrangement is necessary, shown in figure 7.

At intervals along the rim recesses and chambers are constructed. Near the openings of these chambers lay movable plates or rods. When two beings wish to pass, one of them descends into the recess; the other one pushes the rod so as to form a bridge over the opening, walks across it, and then removes the plate so that the one who has descended can get up and go on his way.

If by any chance, while a being is in the recess, the plate or rod which acts as a bridge gets fixed, he is in a dangerous predicament. For suppose a being confined as shown. If he, suffering from want of air, cuts through the roof at *AB*, the whole part to the right of *AB* comes tumbling down. For its only support is severed when *AB* is cut through. It is impossible to make a hole which is not the whole width of matter as it lies on the surface. And with regard to this all constructions have to be made. There cannot be two openings in a wall of a house, unless when the one is open the other is so fashioned as closing to act as a rigid support to the wall, which now depends for its upholding entirely on it.

Thus, in the diagram (fig. 8), the house is held up entirely by the side opposite to the doorway *EF*, which is now open. The roof is sup-

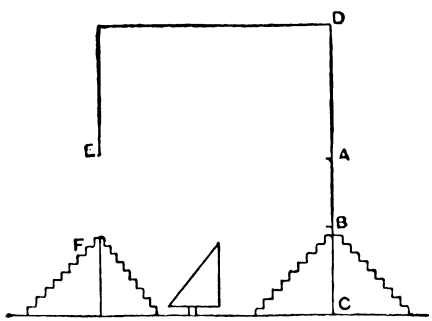


FIGURE 8. A house.

ported by the side *CD*. If an opening *AB* be made in the wall *CD* before the doorway *EF* is closed, the roof will fall in. So, in order to pass through the house, *EF* must be firmly shut up before *AB* is opened. The houses are always built in the interior passages so as to leave the rim of the disk free for locomotion.

And there are many things to be said of the inhabitants on this disk with respect to their social and political life. It is hardly necessary for me to put down much about it here, for any one by using the method of the historian Buckle, and deducing the character of a people from their geographical influences and physical surroundings, could declare what the main features of their life and history must be.

But one or two remarks may be made here. First of all they are characterized by what I may venture to call a crude kind of polarity.

In dwellers in our world this polarity, which shows itself amongst other ways in the distinction of sexes, is tempered and modified.

In every man there is something of a woman, and in every woman there are some of the best qualities of a man.

But in the world of which we speak there is no physical possibility for such interfusion. In a linear existence there would be no consciousness of polarity. It makes its appearance first in the plane, and in a hard and unmitigated form.

It is impossible to do otherwise than caricature these beings when we write of them thus in brief. So let us accept the matter frankly, and, without scruple, look at them in the broadest possible manner.

If the reader will trace and cut out the triangles in the corners of figures 9 and 10, he will obtain four plane beings, two of which are men, two of which are women. The lines down which the cutting is to be made are marked with a black line. Now having cut out two men, whom we will call Homo and Vir, draw a line on a piece of paper to represent the rim of the world on which they stand, and, remembering that they cannot slide over each other, move them about. It must be remembered that the figures cannot leave the plane on which they are put. They must not be turned over. The only way in which they can pass each other is by one climbing over the other's head. They can go forwards or backwards. Much can be noticed from an inspection of these figures. Of course it is only symbolical in the rudest way, but in their whole life the facts which can be noticed in these simple figures are built up and organized into complicated arrangements.

It is evident that the sharp point of one man is always running into another man's sensitive or soft edge. Each man is in continual ap-

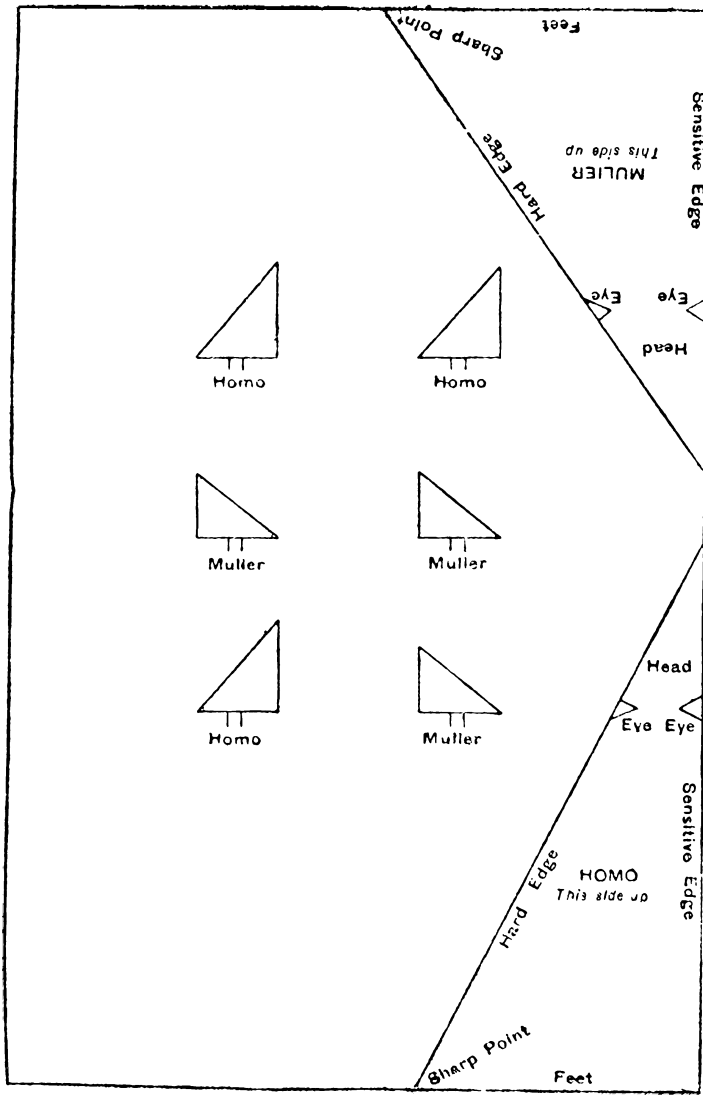

Feet
Sharp Point
Sensitive Edge
MULLER
This side up
Hard Edge
Eye
Eye
Head
Homo
Homo
Muller
Muller
Homo
Muller
Head
Eye Eye
Sensitive Edge
HOMO
This side up
Hard Edge
Sharp Point
Feet

FIGURE 9

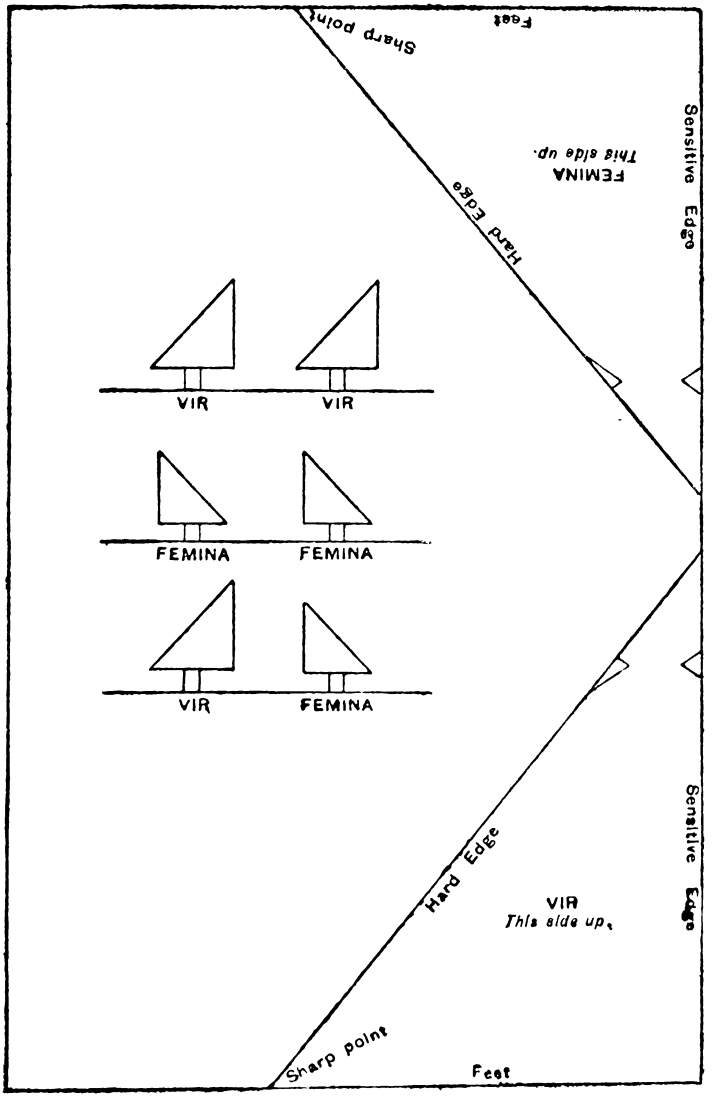

Sharp point
Feet
Sensitive Edge
FEMINA
This side up.
Hard Edge
VIR
VIR
FEMINA
FEMINA
VIR
FEMINA
Sensitive Edge
Hard Edge
VIR
This side up.
Sharp point
Feet

FIGURE 10

prehension of every other man: not only does each fear each, but their sensitive edges—those on which they are receptive of all except the roughest impressions—are turned away from each other.

On the sensitive edge is the face and all the means of expression of feeling. The other edge is covered with a horny thickening of the skin, which at the sharp point becomes very dense and as hard as iron. It will be evident, on moving the figures about that no two men could naturally come face to face with each other.

In this land no such thing as friendship or familiar intercourse between man and man is possible. The very name of it is ridiculous to them. For the only way in which one man can turn his sensitive edge to another man is if one of them will consent to stand on his head. Fathers hold their male children in this way when little, but the first symptom of manhood is connected with a resentment against this treatment.

If now two women, Mulier and Femina, be looked at, the same relation will be seen to hold good between them. By their nature they are predisposed, by accident, to injure one another, and their impressionable sides are, by the very conditions of their being, turned away from each other.

If now, however, Homo and Mulier be placed together, a very different relationship manifests itself. They cannot injure one another, and each is framed for the most delightful converse with the other. Nothing can be more secure from the outside world than a pair of approximately the same height; each protects the sensitive edge of each, and their armoured edges and means of offence are turned against all comers, either in one direction or the other. But, if the pair, through a mutual misunderstanding, happen to be disadjusted, and, their feet on the rim, turn their sharpnesses against one another, they are absolutely exposed to the harms and arrows of the world.

Still, even in this case they cannot wound one another—a happy immunity.

In the annals of this race which I have by me I find a curious history, which, unintelligible for ages to them, admits by us of a simple explanation.

It is said that two beings, the most ideally perfect Vir and Mulier, were once living in a state of most perfect happiness, when, owing to certain abstruse studies of the Mulier, she was suddenly, in all outward respects, turned irremediably into a man. Vir recognized her as the same true Mulier. But she occupied the same position with regard

to him which any other man would. It was only by standing on his head that he could, with his sensitive edge, approach her sensitive edge. She refused to explain how it was, or impart her secret to any one, but she had, she said, undergone a great peril. She manifested a strange knowledge of the internal anatomy of the race, and most of their medical knowledge dates from her. But no persuasion would induce her to reveal her secret; all the privacy of existence would be gone, she said, if she revealed it. She was supposed to have acquired some magical knowledge.

This possession, however, did not make either of them happy, and one day, with fear, she said that she would either die or be restored to the outward semblance of her sex.

She disappeared—absolutely; although she was surrounded by her friends, she absolutely vanished. And had it not been that some days afterwards, cutting through the solid rock for the purposes of some excavations, they accidentally came on a chasm, they would never have found her alive again. For she was found in a cavity in the living rock, warm and beautiful—her old self again.

Her secret died with her.[6]

From our point of view it is easy to see what had happened. If the figure Mulier be taken up and turned over it will be easy to see that, though still a woman, her configuration has become that of a man. To all intents and purposes she is a man. She is rendered incapable of that attitude which is the natural one between the men and women in this land, and the happy relationship between her and Vir is necessarily and entirely broken off. Move about as you will, keeping her figure turned thus on the plane, you will not be able to make her a fitting helpmate for her unfortunate Vir. She must have discovered the secret of raising herself off the surface, and by some accident been turned over. Perhaps she had used this new position to study anatomy—for to an observer thus situated the interior of every body would lie perfectly open—and in prosecuting her studies had overbalanced herself.

I have only mentioned this anecdote, however, for the sake of a curious observation which was made at the time. It was found that when she was in this transformed condition she was absolutely without atmosphere. To explain: ordinarily, apart from anything she said or did, there was a kind of influence proceeding from her which made her

[6] It is evident that Hinton does not share the misogyny which permeates Abbott's *Flatland*, where women are but unreasoning line segments. [R.v.B.R.]

presence agreeable to Vir. When she was turned over she lost this. Now the explanation of this is obvious. To these people light is the agitation of the surface of the bubble; transparent objects are those which do not hinder this agitation in its course. But most bodies and the physical frame of the inhabitants amongst them were not transparent, but stopped and reflected these agitations of the film, thus sending off from their outer edge those vibrations which excited sight in their fellows. But besides these vibrations of light there were finer ones still which were not damped or deflected by the outer edge of the body, but came through the greater part of their frame as if it was transparent. In the interior, however, of their organizations there were certain regions which did arrest these subtler vibrations, and which had (as the eye of light) the power of appreciating them. In connection with these regions there were certain structures, extremely minute, which had the converse power of agitating the film, and so sending forth through the periphery of the body these same minute vibrations. These organs were not of any use, but they formed a sort of means of sympathetic communication between the inhabitants, acting in no very defined way, but certainly producing a sensation of a vague kind. Now when Mulier was turned in the way described, the relation of her frame to the film of the bubble was disarranged, and it was no wonder that this "atmosphere" disappeared.[7]

In many respects the inhabitants of this world are far more advanced than we are, having a simpler problem—how to deal with matter in one plane—they have advanced more nearly to a complete knowledge of its properties. Yet great as their knowledge is, their performance is small. If you but reflect on one single fact, you will see how limited all their efforts must be. *They cannot fix the center of a wheel, so that it rotates round an axis.* For consider a wheel—a small disk lying on their plane. The center on every part of it touches the surface of the bubble on which all things slide freely. To fix this point they would have to drive down into the film—a thing which they cannot do, and which they are far from even imagining.

If they make an opening in the disk they can arrive at the center of it. But then the rod of matter which they put in will prevent the disk from revolving.

The nearest approach to a wheel with a fixed center which they can attain is shown in figure 11, a portion of a circular disk which os-

[7] We have here a nice physical model for the occult notion of personal aura, popularly termed "vibes." [R.v.B.R.]

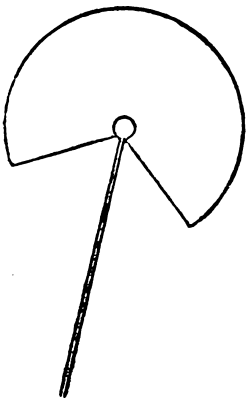


FIGURE 11. The nearest approach to a wheel.

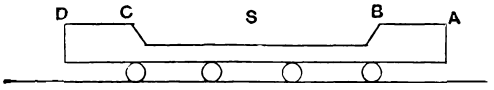


FIGURE 12. A cart.

cillates about the smooth end of a rod built into the substance of the cut-away disk. Their carts are shown in figure 12. They are simply rods placed on rollers: as the rod is pulled along, the rollers turn, and the rod slides along—just as a boat does on the rollers whereby sailors help themselves in hauling it up the beach. As soon as these rollers roll from under the rod, as it goes on in its forward motion, they have to be secured, and then lifted over the cart and put down in front. Thus there are to each cart a set of little disks or rollers, and, as the cart goes on, these rollers have to be lifted over the cart from the back to the front.

There is no means by which this can be made a continuous action. Each roller has to be waited for, lifted separately, and carried over separately. And to put it down in front, the rope by which the cart is dragged along has to be unfastened and fastened up again.

Looking at the figure 12 it will be seen that there is a hollow in the body of the cart. On the part AB the driver sits. In the hollow from B to C is put the load. The load cannot then slip out over the ends of the cart. There is nothing in the cart to prevent it from falling out sideways.

But the contents, as the whole body of the cart, are kept to the smooth surface of the bubble, and are thus supported by it on the side remote from the reader's eye, and also are kept from rising away from this surface by the force of attraction exerted by the film.

Thus the surface of the bubble and its attractive force supply the other two sides of the cart.

But of these two sides, the beings are ignorant, and it seems to them perfectly natural that loads of any kind, even of fluids, should be kept securely in a cart with two ends.

Figure 13 indicates the method by which the rope is fastened to the cart: *C* is the body of the cart; *R* is the rope ending in a wooden step *B*; *A* is an oblong piece of wood. When the rope has to be taken out, *A* is lifted out by its handle, *B* is slipped back and taken out of the recess in *C*, and then the rope is free from the cart. And in a similar way it is secured again.

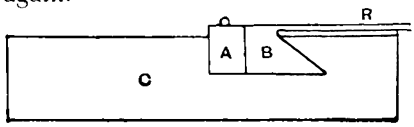


FIGURE 13. How a rope is fixed to a cart.

One very ordinary way of driving machinery with us is by shafting. A long rod is driven round and carries wheels at different places along its length. Now with these inhabitants it was impossible to do this, because the twisting motion round a rod could not be imparted without going out of the thin layer in which they were. Their methods of transmitting motion were by long rods, by a succession of short rods, by pendulums attached to one another, or finally by wheels which drove one another, but which were held by smooth sockets fitting round the rim far enough to steady them, but not so far as to hinder them from touching each other.

As to their science, the best plan is to give a short account of its rise.

They discovered that they were on a disk rotating round an inner center, and also proceeding in a path round the source of light and heat.

They found that they were held in their path by a force of attraction. But this attractive force was not with them as it is with us. With us, since the effect which any particle has on the surrounding particles spreads out in our space if the distance is doubled from a center of attraction, the force it exercises becomes one-quarter of what it was when at the less distance.

With them, however, when the distance doubled, the force of attraction became only one-half of what it was at the less distance. For the light, or attraction, or force of any kind emanating from a particle, only spreads along the film, and does not pass out into the space above or beneath. If they had been on a thick globe instead of a bubble, the laws of attraction would have been the same as with us. But the

bubble on which they were was thin compared with the paths along which the radiant forces spread forth. And thus every force being kept to one plane diminished as the distance from the center of its action (see appendix to this chapter).

Now it was a great problem with them how the light came from the central orb. Their atmosphere, they knew, extended but a small distance above the surface of their disk. And it was quite incapable, moreover, of conveying vibrations such as those of light and heat.

By studying the nature of light they became convinced that to transmit it there must be a medium of extreme rigidity between them and the great source of light.

It is easy enough to see that what they thought was a medium between them and their sun was in reality the rigid surface on which they rested. This elastic film vibrated in a direction transverse to the layer they called matter, and carried the particles of matter with it. But they, having no idea but that the surface on which they were was the whole of space, thought that space must be filled with a rigid medium. They found that the vibrations of the medium were at right angles to the direction in which a ray was propagated. But they did not conceive of a motion at right angles to their plane; they thought it must be in their plane.

It was a puzzle to them how their disk glided with so little friction through this medium. They concluded it was infinitely rare. They were still more puzzled when they had reason to believe it was an opaque substance; and yet that it could be anything else than a medium which filled their space was inconceivable to them. They could never get rid of it from a vacuum, however perfect. Indeed we see that in producing a vacuum they merely cleaned the surface on which they were.

In one respect it might have been advantageous if they had known, for, their law of attraction being what it was, their movement round their sun was not destined to go on for ever; but they were gradually falling nearer and nearer. Now, if only they had made the attempt, they might by some means have got a hold on the surface on which they were, and, by means of a keel which tended to furrow it, have guided their world and themselves in their path round their sun. Indeed, it is possible to imagine them navigating themselves whither they would through their universe—that is, on the surface of their bubble.[8]

[8] This theme is developed at length in *An Episode of Flatland.* [R.v.B.R.]

It was also unfortunate in another respect that they did not realize the fact of the supporting surface, for the feeling which they came to have of being suspended in space, absolutely isolated, was a very unsettling one, and tended to cause in them a certain lack of the feeling of solidarity with the rest of the universe.

We have seen that their laws of mechanics were very different from ours. But they had after all an experience of our mechanical principles, though in a curious way. In all motions of any magnitude moving bodies were confined to the surface of the plane. But where the small particles were concerned there was more liberty of motion. The small particles were free in their movement; although they could not go more than a very small distance away from the film on which they rested, still they were capable of motion perpendicular to it. Thus a long line of particles connected together could rotate as a whole, keeping straight like a twisting wire, and by means of many strings of particles thus connected, movements could be transmitted in a way which was totally unlike the mechanical movements to be seen in the case of large masses.

This motion of rotation round an axis lying in the plane was to them what electricity is to us. It was quite a mysterious force. But it was extremely useful in its applications. Having no idea of a rotation which in taking place went out of their surface, they could not conceive a reason for the results of such movements.

It can easily be seen how many kinds of forces they could have. There was the spinning motion of the small particles on the surface. This they were aware of—it produced many appearances, but it was not fitted for transmission across great distances, as each particle was apt to be hindered in its rotation by its neighbor. Sometimes, however, when conditions were favorable, many of these rotations were harmonious, and waves were produced in their matter resembling the waves in our ocean.

There were only two other kinds of motion. One was an up and down vibration of the film carrying matter with it; the other was the twisting of strings of particles which were rigidly connected together. The up and down motion of the film was to them light. Those kinds of matter which did not hinder this motion were said to be transparent; those kinds which, lying on the film, hindered the motion or threw it back were said to be opaque.

The twisting motion round an axis was to them what electricity is to us. And when this twisting motion in one direction or another was

conveyed to the particles of small masses which were free to move, many curious effects were produced analogous to the movements of electrified bodies. There are obviously no other rotations or vibrations possible; hence in that world there is nothing corresponding to magnetism. Their light was simple, and could not be split into two kinds as our light can be—into two kinds of polarized light.

Was there no sign, then, by which the inhabitants of this world could gain a knowledge of their own limitation? There was. There was both a sign and the interpretation of it lying before them. They knew that they could have two triangles precisely similar, and yet such as could not be turned the one into the other by any movement in the plane. How two things could be so alike, and yet differ in some mysterious way, was to them a puzzle. As an instance of such triangles may be taken those used in figure 9 to represent the man and the woman. They may be exactly equal, yet the beings in a plane world cannot turn them so that one would coincide with the other.

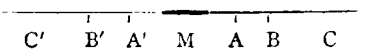


FIGURE 14

Yet had they but considered the case of a being lower in the scale of space existence than themselves, they would have seen the answer to their riddle. For consider a being confined altogether to a line (fig. 14). Let M be the being, and let him observe the three points ABC, and let him form an idea of them and their positions with regard to each other, which he measures by the distance he has to travel to reach one after passing the other.

Let him also become aware of the three points $A'B'C'$, forming a precisely similar set on the other side of him.

It may be objected that the being in the line could not conceive any point lying beyond A, but that his experience would be limited to the points A and A'. If A and A' are material particles this would be the case, but we may suppose them to be places in the line marked out by cold and heat, or some such means. Then a being could conceive a series of positions in his space such as A, B and C; A', B', C'.

If now he remembers each set, and thinks about them, he finds that they are alike in every respect. But he cannot make them coincide with each other. For if he pushes the set ABC along the line, when AB and $A'B'$ are together C is just where it ought not to be. It is not on C'. And if he gets C on C', then AB has gone far away.

He would neither be able to make them coincide nor to conceive their coincidence.

There would be no movement within the realm of his experience which would make them coincide.

Yet the dweller in a plane world could easily make these sets of points coincide, for he would bend the whole line round in his plane so that A coming on A', B should come on B', and C on C'. There would be no difficulty to him in doing this. And he does it in virtue of there being to him a movement possible which is not possible to the being in the line. He has a liberty of motion unknown to the linear being.

And now why should he not reason thus, "Something which to the linear being is inconceivable, to me is conceivable. Then may not things inconceivable to me be yet possible? May it not be possible that two triangles which are like one another, but yet which cannot be thought by me as coinciding—may not these triangles be able to be made coincident?"

In this simple fact of his perpetual observation was really the proof of the whole matter if he had but looked at it, the sign manual of his limitation, the promise of his liberation from it in thought, the key to the explanation of the mysterious minute actions by which he was surrounded, and perchance a help to the comprehension of a higher life.

Appendix

In our world a particle of matter which sends forth influence on the surrounding matter does not send its radiant energy off along a plane, but from the particle all the influence spreads out into space. And the most convenient instance in our world to consider is that of a luminous point from which rays spread out in every direction. Let M in figure 15 be such a point—a particle of matter sending forth luminous rays in our three-dimensional space.

Instead of studying how these rays spread out in every way all around M, let us only consider those which, passing out from M, fall on the square $ABCD$. $ABCD$ casts a shadow, and this shadow extends, and is found to be bigger the further off from M it is measured. Suppose, at the distance from M, ME, we put a square in the path of the shadow so as just to receive the shadow on it exactly. Let $EFGH$ be this square. As is shown by the dotted lines, this square

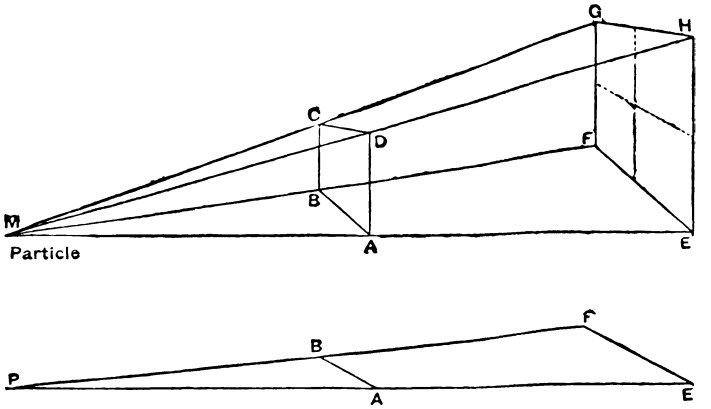


FIGURE 15. Particles in space and in a plane exerting force.

will be four times as large as the square *ABCD*. So when the distance is doubled, the shadow becomes four times as big.

Now those rays of light which fall on *ABCD* would, if they were not interrupted by it, spread out so as to exactly cover *EFGH*. Thus the same amount of light which falls on the small square *ABCD* would, if it were taken away, fall on the large square *EFGH*.

Now since the large square is four times the size of the small square, and the same amount of rays fall on it—for it only receives those which would fall on the small square—there must be at any part of it an illumination one-quarter as strong as there would be at any point on the small square.

Thus the small square, if placed in its position, would seem four times as bright as the large square.

Thus, when the distance from the origin of light is doubled, the amount of light received by a surface of given area becomes one-fourth of what it was at the less distance.

This is what is meant by varying inversely as the square of the distance. When the distance is doubled the intensity of the light is not simply less, but is halved and halved, and becomes one-quarter of its previous intensity.

But in the case of a particle resting on a thin sheet of metal, and shaking the metal—as, for instance, a metal plate can be made to shake by a violin bow—then this law would not hold.

Take the second figure. Let *P* be the particle, and let the influence proceeding from it fall on the rod *AB* lying on the plane, and let us

suppose the rod to stop the vibrations from going beyond it, to receive them and to turn them back just as a body does the light. Then the "shadow" of AB would spread out away from P; and if another rod EF were put in at the distance PE, which is double of PA, then, to exactly fit the shadow, it would have to be double the length of AB; and the vibrations which fell on AB would exactly fall on EF. Now since EF is twice as long as AB, the vibrations which fall on any part of it will be one-half as intense as the vibrations which fall on a portion of matter of the same size lying where AB lies.

Thus in a plane the influence or force sent out by any particle would diminish as the distance. It would not "vary inversely as the square of the distance," but would "vary inversely as the distance."

A PICTURE OF OUR UNIVERSE

[This selection contains excerpts of "A Picture of Our Universe" from *Scientific Romances*, Vol. 1 (1884). It includes material from Chapters 1 and 3, and Appendix 1.]

The Kilkenny Cats

It seems to me that the subject of higher space is becoming felt as serious, and fraught with much that is of the deepest interest, not only as a scientific problem, but in other ways also. It seems also that when we commence to feel the seriousness of any subject we partly lose our faculty of dealing with it. The intellect seems to be overweighted somehow, and clogged. Perhaps the suppositions we make seem to us of too great importance, and we are not willing enough to let them go, fearing to lose the thing itself if we lose our hold of the means by which we have first apprehended it.

But whatever may be the cause, it does seem undoubtedly the fact that the mind works more clearly and more freely on subjects which are of slight importance.

And I propose, that without ignoring the real importance of the subject about which we are treating, we should cast aside any tension from our minds, and look at it in a light and easy manner.

With this object in view let us contemplate a certain story which bears on our problem.

It is said that once in a certain region of Ireland there took place a curious contest. For in Kelkenny there were two cats so alike in size, vigour, determination, and prowess, that, fighting, they so clawed, scratched, bit, and finally devoured each other, that nothing was left of either of them save the tail.

Now, on reflecting on this story, it becomes obvious that it originated when looking-glasses were first imported into Ireland from Italy. For when an Irishman sees for the first time anything new, he always describes it in an unexpected and yet genial and interesting manner. Moreover, we all know what contentious fellows they are,

and how all their thoughts run on fighting. And I think if we put this problem to ourselves, how by bringing in fighting to describe a looking-glass, we shall see that the story of the Kilkenny cats is the only possible solution. For consider evidently how it arose. Depositing his favorite shillaly in a corner, the massively built Irishman, to whom the possession was a novelty, saw reflected in his looking-glass the image of his favorite cat. With a scrutinizing eye he compared the two. Point for point they were like. "Begorra if I know which of the two would win!" he ejaculates. The combat becomes real to him, and the story of the Kilkenny cats is made.

Now, to our more sober mind, it is obvious that two cats—two real material things—could not mutually annihilate each other to such an extent. But it is perfectly possible to make a model of the Kilkenny cats—to see them fight, and to mark the issue.

And I propose to symbolize or represent the Kilkenny cat by a twist. Take a pencil, and round it twist a strip of paper—a flat spill will do. Now, having fastened the ends on to the pencil by two pins, so that it will not untwist, hold the paper thus twisted on the pencil at right angles to the surface of a looking-glass: and in the looking-glass you will see its image. In figure 16, *M* represents the mirror, and on the left hand is shown the twist, on the right hand the image twist.*

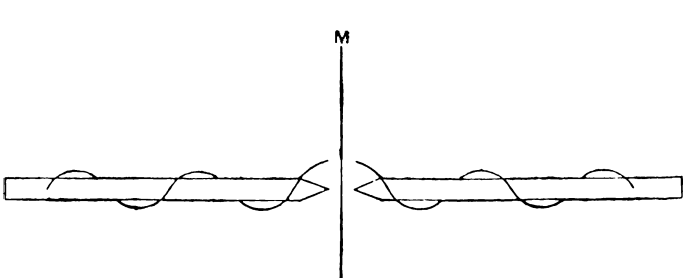


FIGURE 16

Now take another pencil and another piece of paper, and make a model of what you see in the glass. You will be able to twist this second piece of paper in a spiral round this second pencil so that it is an exact copy of what you see in the glass. Now put the two pencils together end to end, as they would be if the first pencil were to ap-

* Hinton's use of the word "image" is akin to our modern expression "mirror image."

proach the glass until it touched it, meeting its image: you have the real copy of the image instead of the image itself. Now pin together the two ends of the pieces of paper, which are near together, and you have your two Kilkenny cats ready for the fray. To make them fight (remember that the twists—not the paper itself, but the paper twisted—represent the cats), hold firmly and pull the other ends (the tail ends, so to speak), so as to let each twist exercise its nature on the other.

You will see that the two twists mutually annihilate each other. Without your unwrapping the paper the twists both go, and nothing is left of them.

Now the image of the twist as a real thing was made by us. It did not exist in nature other than as a mere appearance.

But I want you to imagine this process of producing a real image as somehow existing. I want you to lay aside for the present the question of how it could be done, and to conceive twists and image twists.

This is the mechanical conception I wish you to adopt—there are such things as twists. Suppose by some means to every twist there is produced its image twist. These two, the twist and its image, may exist separately; but suppose that whenever a twist is produced its image twist is also produced, and that these two when put together annihilate each other.

With this conception let us explore the domain of those actions which are called electrical.

We will suppose that a metallic body consists of particles so arranged together that it easily acts as a set of minute threads or chains of particles which will twist, each thread or chain twisting as a whole. Thus the conception which should be formed of a metallic body conducting electricity along it is this: Conceive a bundle of very fine but very rigid wires, each wire twisting separately but with the same kind of twist as all the others, and each, as it twists, rotating amongst its fellow threads. If we have a metal rod we can twist it between the finger and thumb. This is not the kind of twist we suppose, but that each separate string of particles is thus twisted, so that each set twisting remains in the same part of the metal rod—but is turning round in its fixed position. This is a body conveying an electric current. If the current will not pass, the set of minute wires must be conceived as held at the far end, and given a twist, starting from the point where the electricity is communicated. Now if a conductor is thus charged and left, it is found that it retains its charge; to be discharged it must be touched with another conductor. Hence this twist of minute threads

differs from a twist of a wire in that the threads cannot untwist of themselves unless other threads come into contact with them to which they can impart the twist. That this should be the case may depend on the fact that the twisting strings are strings of molecules, and the ends of them would thus be connected with other molecules with something of the same tenacity as that with which the strings themselves cohere together, and are unable to unlock themselves from these insulating or untwisting molecules.

We have seen that if we make a certain supposition as to the calling up of an image twist by a twist of molecular matter, then the main facts of electricity are capable of an explanation, which, involving merely the motion of ordinary matter, is far preferable to the idea of there being a mysterious fluid, and more in harmony with our present ideas of electricity.

And yet it is impossible to retain this supposition unless a clear mechanical explanation can be given of how a real image of itself can be called up by the twist which we suppose electricity to be.

We can by intelligent agency produce a twist which is the real image of a given twist. But it would be absurd to suppose amongst the molecules an agency which, acting with prescribed aim, gave in that domain those real simulacra, those evident images, those phantoms with which we in our larger world of masses are for ever mocked.

And yet it would be curious if such an hypothesis were to claim a recognized position in our mental apparatus with which we think about nature.

For in that molecular world, if we imagine it to ourselves, there would be a curious state.

If we consider a twist and its image, they are but the simplest and most rudimentary type of an organism. What holds good of a twist and its image twist would hold good of a more complicated arrangement also. If a bit of structure apparently very unlike a twist, and with manifold parts and differences in it—if such a structure were to meet its image structure, each of them would instantly unwind the other, and what was before a complex and compound whole, opposite to an image of itself, would at once be resolved into a string of formless particles. A flash, a blaze, and all would be over.

To realize what this would mean we must conceive that in our world there were to be for each man somewhere a counter-man, a presentment of himself, a real counterfeit, outwardly fashioned like

himself, but with his right hand opposite his original's right hand. Exactly like the image of the man in a mirror.

And then when the man and his counterfeit met, a sudden whirl, a blaze, a little steam, and the two human beings, having mutually unwound each other, leave nothing but a residuum of formless particles.

The Aether

There are some expressions which, being somewhat vaguely used, are apt to cause confusion in the mind of those who read or hear about higher space.

And perhaps the most mischievous is the expression, a curvature of space. Now of space as it is generally used, in its accepted significance, there can be no curvature. For space means a system of positions extending uniformly in the number of dimensions we choose to fix upon.

If we take the straight line as our space, we may call it 1 space; then the set of positions follow one on after the other without bending. If the line is bent it becomes a *line,* not a straight line. It should not be called 1 space, but a thing in 2 space. That is, it is a bent line in a plane.

A being who was on the line might not perceive the fact of this bending, and it might not affect the measurements he made. But if the line ran into itself again, and he found that he was moving on what we should call a circle, this would in no way affect his idea of space. He would recognize that what he called space, namely, his line, was not space, but a curved thing in 2 space.

Similarly, taking a plane—this is by definition not curved in any way, known or unknown, and it can only be conceived to be bent by ampler space being conceived, and its being imagined as having force applied to it so as to become a bent thing in this ampler space. In this case the term "plane" is not the correct name.

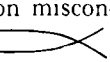
And so about our three-dimensional space; we cannot be robbed of that idea, although it might conceivably be proved that our earth and our whole universe were on a curved thing in 4 space.

We will then keep the term "space" for the ordinary conception; and call it 1, 2, 3, 4 space, according to the number of supposed independent directions.

A curved line or surface or solid we will call a 1, 2, or 3 thing, according to the number of dimensions in it.

A straight line is a 1 thing possible in 1 space. A circle is a 1 thing possible in 2 space. At any point of it a being in it is limited to motion in one direction, while the circle itself involves two dimensions. The surface of a sphere is a 2 thing possible in 3 space. The rind of an orange, or the orange itself, is a 3 thing possible in 3 space.

It will be observed that the surface of the sphere, although only a 2 thing, involves the conception of 3 space, and cannot be understood without the use of the idea of 3 space. It is a 2 thing because at any point of the surface a being can only move in two independent directions. A crooked line drawn on the surface of a sphere is a 1 thing in a 2 thing in 3 space.

Another very common misconception is occasioned by the use of a figure of this kind  to represent a "knot" in 2 space.[9]

It obviously corresponds in 2 space to an iron rod welded together at the crossing place of the loop, so that it is indistinguishable which is the one free end, which the other. At the crossing point the two lines represented by the two ink marks must be absolutely one and the same.

If one line be supposed to go over the other, by however small a distance, it would leave the plane. It would suddenly become invisible to the creature in the plane, and it would appear again at the other side of the line it crossed as if it came from nowhere.

It would be as extraordinary a sight as if we saw a pole going up to a brick wall, then beyond the brick wall the rest of the pole appearing—not going through the brick wall, nor coming round it—but somehow appearing; part of the same pole moving when it moved, obviously connected with it, and yet with no joining part which we could possibly discover.

Again, it sometimes appears to be thought that the fourth dimension is in some way different from the three which we know. But there is nothing mysterious at all about it. It is just an ordinary dimension tilted up in some way, which with our bodily organs we cannot point to. But if it is bent down it will be just like any ordinary dimension: a line which went up into the fourth dimension one inch will, when bent down, lie an inch in any known direction we like to point out. Only if this line in the fourth dimension be supposed to be connected rigidly with any rigid body, one of the directions in that rigid body must point away in the fourth dimension when the line that was in the fourth

[9] Hinton is probably referring here to the book *Transcendental Physics*, by Johann Zöllner, which was first published in 1879. In this book Zöllner makes just the mistake which Hinton refers to here. [R.v.B.R.]

comes into a 3-space direction.

If the reader will refer back to the chapter on the plane world he will find a description of the means by which a being there might know that he was in a limited world, and that his conception of space was not of what was really the whole of space, but of the limited portion of it to which he was confined by his manner of being.

The test by which such a being could discover his limitation was this. He found two things, each consisting of a multitude of parts—two triangles; and the relationship of the parts of the one was the same as the relationship of the parts of the other. For every point in the one there was a corresponding point in the other. For every pair of points in the one there was a corresponding pair of points in the other. In fact, considered as systems made up of mutually related parts, each was the same as the other.

Yet he could not make these two triangles coincide.

Now this impossibility of bringing together two things which he felt were really alike was the sign to him of his limitation; and by reflecting on the similar appearance which would present itself to a being limited to a straight line—by thinking of two systems of points which were really identical, and which he could make coincide, but which a line being could not make coincide, he would be led to conclude that he in his turn was subject to a limitation.

Now is there any object which we know which, considered as a whole consisting of parts, is exactly like another whole, the two having all their parts similarly arranged, so as to form in themselves two identical systems, and yet the one incapable of being made to coincide with the other, even in thought?

Let us look at our two hands.

They are (except for accidental variations) exactly alike. And yet they cannot be made to coincide.

And here, if we reflect on it, is the sign to us that we are limited in our notions of space—that we are really in a four-dimensional world.

Watching a ship as it recedes from the shore we see that it becomes hull down before it vanishes, and know that the earth is round. And no less certainly do our two hands, in their curious likeness and yet difference, afford to us a perpetual proof of our limitation, and indicate a larger world.

This sign really tells us more than the mere fact of our limitation: it tells us where to look for the possibility of four-dimensional movements. It tells us that movements of any degree of magnitude relative to us are not possible in the fourth dimension. It tells us to look for

four-dimensional movements in the minute particles of matter, not in the movements of masses of about our own size.

The task before us is difficult. We have to make up from the outside what the appearances of a higher space existence are to us in our space, and then we have to look at the facts of nature and see if they correspond to these appearances.

Let us take a few isolated points and look at them patiently.

To a being standing on the rim of a plane world a straight line absolutely shuts out the prospect before him. If the straight line is infinite it cuts his world in two; he can never hope to get beyond it.

It is to him what an infinite plane would be to us, stretching impassably in front of us, cutting us off from all that lies on the other side.

But we know that a point can move round this line. It can revolve round it by going out of the plane, and coming down again into the plane on the other side of the line.

This movement would be inconceivable to a plane being; for he can only conceive it possible to get to the other side of the line by going to the end of it and coming back along the other side of the line.

Now take a piece of paper and put a dot right in the middle and suppose that it has no means of passing through the paper. We can only conceive the dot getting to the other side of the paper by passing round the edge and coming back again to the position underneath where it was.

But by a four-dimensional movement it can slip round the paper without going to the edge.

A set of words may help. In a plane a body rotates round a point—rotation takes place round a point. In space rotation is always round a line—the axis. In four-dimensional space rotation takes place round a plane.

To take a farther consideration of this point—a plane being can see one side or the opposite of a straight line. He can only see it in one direction or in the reverse direction. But we can look at a straight line from a direction at right angles to that in which a plane being looks at it. We can look at a straight line from points which go all round it.

Similarly, a being in four-dimensional space can look at a plane from a direction at right angles to that in which we look at it. If we try to think of this we shall imagine ourselves looking at the thin edge. But this is not what a four-dimensional being would mean. He would

see the plane exactly as we see it, but it would be from a direction at right angles to that in which we look.

In working with four-dimensional models it is a curious sensation until we become used to it—that of looking at a plane at one time, and then looking at it again; and, although it seems just the same—as square in front of us as before—realizing that we are looking at it from a direction at right angles to that of our former view.

And in four dimensions a point which is quite close to a plane can revolve round it without passing through it, thus presenting to us the appearance of vibrating across the plane, but not passing through it.

The appearance is as wonderful to us as it would be for a plane being to see a point which was in front of a line quickly passing behind it without having gone round the end. Such a point would appear to the plane being to vibrate across his line without passing through it.

Now if we stand in front of a mirror we see the image of ourselves. If we were to go round the mirror and take behind it the position which our image seemed to occupy, we should not be able to make ourselves coincide with it. In the mirror opposite to our left hand is the image of our left hand; but if we passed round, our right hand would be in the place in which we imagined we saw the image of our left hand. And thus we cannot make ourselves coincide with our image. But by a rotation in four-dimensional space we could put ourselves so as exactly to coincide with our image. This can be seen by referring to the case of the straight line, figure 17.

Let ABC be a triangle, and G a line. If ABC moves round the end of the line, it can take up the position $A'B'C'$; but it cannot anyhow be made to take the position shown in figure 18, $A'B'C'$.

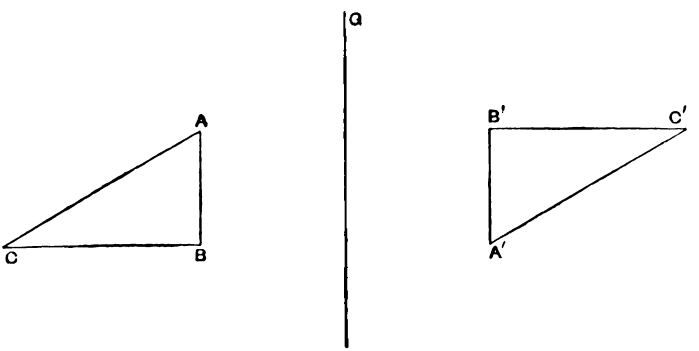


FIGURE 17

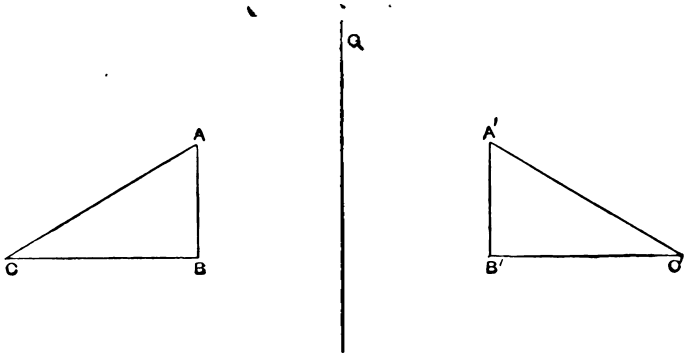


FIGURE 18

But if we move the triangle ABC out of the plane round the line G as axis, it will, in the course of its twisting round this axis, come into the position $A'B'C'$. It will come into this position when it has twisted half-way round. The point A, for instance, twists round in a circle lying in a plane which contains the direction A to A', and the direction at right angles to the paper. Twisting half-way round in this circle, it becomes A', and so on for the other points. Now a being who did not know what a direction was which lay out of the plane would not be able to conceive this twisting and turning movement. It would be as impossible for him to conceive the triangle ABC turned into the triangle $A'B'C'$, as it would be for us to suppose ourselves turned into the looking-glass image of ourselves by a simple twisting.

Yet just as a thing inconceivable to the plane creature can be done, so we could be twisted round and turned into our image. But this only holds theoretically; our relation to the aether is such that we cannot be so turned, or any bodies of a magnitude appreciable to our senses.

If we consider the case of a being limited to a plane, we see that he would have two directions marked out for him at every point of the rim of matter on which he must be conceived as standing. This is up and down, and forwards and backwards—the up being away from the attracting mass on which he is.

Now, if he were to realize that he was in three-dimensional space, but confined to a plane surface in it, his first conclusion would be that there was a new direction starting from every point of matter, and that this new direction was not one of those which he knew. This new direction he could not represent in terms of the directions with which he was familiar, and he would have to invent new terms for it.

And so we, when we conceive that from every particle of matter there is a new direction not connected with any of those which we know, but independent of all the paths we can draw in space, and at right angles to them all—we also must invent a new name for this new direction. And let us suppose a force acting in a definite way in this new direction. Let there be a force like gravitation. If there is such a direction, there will probably be a force acting in it; for in every known direction we find forces of some kind or another acting. Let us call away from this force by the Greek word *ana,* and towards the center of this force *kata.* Then from every point in addition to the directions up and down, right and left, away from and towards us, is the new direction ana and kata.

Now we must suppose something to prevent matter passing off in the direction kata. We must suppose something touching it at every point, and, like it, indefinitely extended in three dimensions.

But we need not suppose it—this unknown—to be infinitely extended in the new direction ana and kata. If matter is to move freely, it must be on the surface of this substratum. And when the word surface is used it does not mean surface in the sense that a table top is a surface; it is not a plane surface, but a solid space surface. If from every point of a material body a new direction goes off, the matter which fills up the space produced by the solid moving in this new direction will have the solid it started from as its surface, and will be to it as a solid cube is to the square which bounds it on the top.

Now this body which extends thus, bearing all solid portions of matter in contact with its surface by every point of them, may be thick in the kata direction or thin.

If it is thick, then the influence of any point streaming out in radiant lines will pass as in all space directions, so out also in this new direction.

And then if its influence spreads out in this new direction, its effect on any particle near it will diminish as the cube of the distance; for, besides filling all space, it will have also to fill space extended in this new direction.

But we know that the influence proceeding from a particle does not diminish as the cube of the distance, but as the square of the distance.

Hence the body which, touching all solid bodies by every point in them, and supports them extending itself in the kata direction—this body is not thick in this direction, but thin. It is so thin that over

distances which we can measure the influence proceeding from a body is not lost by spreading in this new kind of depth.

Thus the supporting body resembles, as far as we know it, a portion of a vast bubble. But moving on the surface of this bubble we can pass up and down, near and far, right and left, without leaving the surface of the bubble. The direction in which it is thin is in a direction which we do not know, in which we cannot move. But although we cannot make any movements which we can observe with our eyes in this direction, still the thin film—thin though infinitely extended in any way which we can measure—this thin film vibrates and quivers in this new direction, and the effects of its trembling and quivering are visible in the results of molecular motion. It only affects matter by its movement in directions at right angles to any paths which we can point to or observe, and these movements are minute; but still they are incessant, all-pervading, and the cause of movements of matter. It is smooth—so smooth that it hinders not at all the gliding of our earth in its onward path. Hence it does not transmit a direct pull or push in any direction from one particle to another; but by the twistings and vibrations of the material particles it is affected, and conveys from one to another these movements. Yet to bear up all matter, and thus hold it on its vast solid surface, it must be extremely rigid and unshatterable; and hence it cannot be permanently altered or twisted by any force proceeding from matter; but receiving from matter any push or twist, it is impressed with it for some distance; then, reasserting itself, it produces an image displacement or twist, and this image it transfers to the particles of matter which it touches.

Sometimes, as when light comes from the sun, this displacement and image is repeated and repeated innumerable times before at last we, receiving it, become aware of the origin of the disturbance.

But the properties and powers of this solid sheet—this film quivering and trembling, yet infinite and solid—are too many to begin to enumerate. The aether is more solid than the vastest mountain chains, yet thinner than a leaf; undestroyed by the fiercest heat of any furnace, for the heat of the furnace is but its shaking and quivering; bearing all the heavenly bodies on it, and conveying their influence to all regions of what we call space.

And by some mysterious action it calls up magnetism from electricity; by its different movements it gives the different kinds of light their being.

Of itself untrammelled and unclogged by matter, it vibrates and shakes with the speed and rapidity of the vibrations of light. But when matter lies on it—when air, even in its rarest condition, lies on it—its proper movement is damped and some of its quick shakings that are light, slow down to the obscure vibrations of heat. Thus of itself it will not take up the vibration of a hot body, but selects only those orbs which are glowing with radiant light wherefrom to take its thrilling messages. But when matter lies on it, it takes obediently the less vivacious movements of terrestrial fires.

A being able to lay hold of the aether by any means would, unless he were instantly lost from amongst us by his staying still while the earth dashes on—he would be able to pass in any space direction in our world. He would not need to climb by stairs, nor to pass along resting on the ground.

And such a being, even as thin as ourselves, and as limited, if not even in physical powers, but merely in thought he became aware of his true relation to the aether, he would see all things differently.

From all shapes would fall that limitation of thought which makes us see them differently to what they are; and in largeness and liberty of possible movement his mind would travel where ours but creeps, and soar and extend where ours journeys and diverges.

It is impossible in contemplating the rudiments of four-dimensional existence to prevent a sense of largeness and liberty penetrating even through the profoundness of our ignorance.

Whether we shall find beings other than ourselves, when we have explored this larger space, cannot be said.

But there is a path which holds out a more distinct promise.

When the conditions of life on a plane are realized it becomes evident that much of that which is to us merely natural—obvious from the very conditions of our life—could only be attained by beings on the plane as the result of artificial contrivances and modifications of their natural tendencies. In their progress and development they would, as it were, represent on the plane the features of the normal and undeveloped life of three-dimensional beings, and they would attain, as a result of moral labor and energy, a position which children in our higher life are born to without trouble or thought.

And so we in our advancing civilization may to the eyes of some higher beings represent in our arrangements and institutions an approach to the simplest matters of fact in their existence. We are

separated from such a view by our bodily conditions, but we are not to be prevented from taking it with our minds.

By building up the conception of higher space, by framing the mechanics of such a higher world, we may arrive at a fairly accurate knowledge of the conditions of life in it.

And then, with that element in our thought, with the reasoned-out characteristics present to our minds of what life on a higher physical basis would be, we may be able to judge amidst conflicting tendencies with more certainty and calmness.

In one of the following papers of this series an account will be given of some of the facts which we can discern about the machinery and appliances of four-dimensional beings.[10]

But the work of real discernment belongs to those who will from childhood be brought up to the conception of higher space.

Appendix

A supposition can be made with regard to the aether which renders clearer an idea often found in literature.

This idea is that of the freedom of the will. If the will is free, it must affect the world so as to determine chains of actions about which the mechanical laws hold true. We know that these mechanical laws are invariably true. Hence, if the will is an independent cause, it must act so that its deeds produce to us the appearance of a set of events determined by our known laws of cause and effect. The idea of the freedom of the will is intimately connected with the assertion that apparent importance, command of power, greatness and estimation, are outside considerations, not affecting the real importance and value of any human agent. These ideas can easily be represented using the idea of the aether as here given.

For suppose the aether, instead of being perfectly smooth, to be corrugated, and to have all manner of definite marks and furrows. Then the earth, coming in its course round the sun on this corrugated surface, would behave exactly like the phonograph behaves.

In the case of the phonograph the indented metal sheet is moved past the metal point attached to the membrane. In the case of the earth it is the indented aether which remains still while the material earth

[10] Hinton makes good on this promise in Chapter 6 of *The Fourth Dimension*, where he considers at some length the questions, "What would a wheel be in four dimensions? What the shafting for the transmission of power which a four-dimensional being would use?" [R.v.B.R.]

slips along it. Corresponding to each of the marks in the aether there would be a movement of matter, and the consistency and laws of the movements of matter would depend on the predetermined disposition of the furrows and indentations of the solid surface along which it slips.

The sun, too, moving along the aether, would receive its extreme energy of vibration from the particular region along which it moved, and the furrows of the intervening distance give the phenomena actually observed of our relationship to the sun and other heavenly bodies.

Thus matter may be entirely passive, and the history of nations, stories of kings, down to the smallest details in the life of individuals, be phonographed out according to predetermined marks in the aether. In that case a man would, as to his material body, correspond to certain portions of matter; as to his actions and thoughts he would be a complicated set of furrows in the aether.

Now what the man is in himself may be left undetermined; but he would be more intimately connected with the aether than with the matter of his body. And we may suppose that the aether itself is capable of movement and alteration; that it moulds itself into new furrows and marks.

Thus the old woman smoking a pipe by the wayside years ago, and whom I somehow so often remember, is not much different from me—we are both corrugations of the same aether.

Now our consciousness is limited to our bodily surroundings. Yet it may be supposed that in an action of our wills we, whatever we are (and for the present let us suppose that we are a part of the aether), we may be altering these corrugations of the aether. A single act of our wills, when we really do act, may be a universal affair with quite infinite relations. Thus it may be the immediate presentation to us of an alteration proceeding from us of all that set of corrugations which represents our future life; it may be the whole disposition and lie of events, which are prepared for the earth to phonograph out, being differently disposed. And it evidently is quite independent of the particular furrows in which such alteration first occurs. That long strip of aether which is a very humble individual may, by an act of self-configuration, affect the neighboring long strips and produce great changes. At any rate the intrinsic value of the will is quite independent of the kind of furrows along which any material human body is proceeding.

CASTING OUT THE SELF

[This selection contains excerpts of "Casting Out the Self" from *Scientific Romances,* Vol. 1 (1884).]

The words which I have chosen as the title of this paper are the expression for a process which has been asserted to be one that occurs alike in our mental and in our moral life. It has so happened that in certain of my own inquiries I have applied this process; and the details may be of interest. But I must warn the reader not to expect any wide views on life, or far-reaching thoughts, or any of the warmth of human affairs. What I think about is Space; and it is the application of the principle of casting out the self in attaining a knowledge of Space about which I have something to say.

And, firstly, as a bit of absolute human experience is never without value, but that which we make up is often so, I may as well cast the fear of ridicule aside and enable the reader to take in, in a few lines, the exact commencement of my inquiry.

The beginning of it was this. I gradually came to find that I had no knowledge worth calling by that name, and that I had never thoroughly understood anything which I had heard. I will not go into the matter further; simply this was what I found, and at a time when I had finished the years set apart for acquiring knowledge, and was far removed from contact with learned men. I could not take up my education again, but although I regretted my lost opportunities I determined to know something. With this view I tried to acquire knowledge in various ways, but in all of them knowledge was too impalpable for me to get hold of it. And I would earnestly urge all students to make haste in acquiring real knowledge while they are in the way with those that can impart it; and not rush on too quickly, thinking that they can get knowledge afterwards. For out in the world knowledge is hard to find.

At length I came to find that the only thing I could know was of this kind. If, for instance, there were several people in a room, I could not

know them themselves, for they were too infinitely complicated for my mind to grasp; but I could know if they were at right or left hand of one another, close together, or far apart. And the same of, to take another instance, botanical specimens in a book. I could not grasp the specimens—each was too infinitely complicated, and each part too infinitely complex—but I could tell which specimen was next which.

Accordingly, being desirous to learn something thoroughly, and since, in the arrangement of any different objects, there was such a lot of ignorance introduced by the objects being different—each bringing in its own ignorance and feeling of bewilderment—I determined to learn an arrangement of a number of objects as much alike as possible.

Accordingly I took a number of cubes, which were as simple objects as I could get, arranged them in a large block, and proceeded to learn how they were placed with regard to each other. In order to learn them I gave each of them a name. The name meant the particular cube in the particular position.

Thus, taking any three names, I could say, about the three cubes denoted, how they were placed with regard to one another: one, say, would be straight above the first with four intervening, the third would touch the second on the right hand, or some similar arrangement.

Now in this way I got what I conceived to be knowledge. It was of no use or beauty apparently, but I had no reason to use it or to show it.

It is about this bit of knowledge that I want to speak now—a block of cubes, and the cubes are known each one where it is.

Sometimes I have been tempted to call this absolute knowledge, but have been reminded that I did not know the cube itself. Against this I have argued. But in argument we say many things which we do not understand, and my conclusion is, on the whole, that the objection is well founded. Still, if not knowledge absolute, the knowledge of this block approaches more nearly to knowledge absolute than any other with which I am acquainted, because each cube is the same as its neighbor, and instead of an arrangement of all sorts of diverse ignorances we have only one kind of ignorance—that of the cube. Each of the cubes was an inch each way, and I learnt a cubic yard of them. That is to say, when the name of any cube was said, I could tell at once those which it lay next to; and if a set of names were said, I could tell at once what shape composed of cubes was denoted. There were

216 primary names, and these, taken in pairs, were enough to name the cubic yard.[11]

For the practical purpose of this paper, however, it will suffice if the reader will imagine a block of twenty-seven cubes, forming a larger cube, each cube being denoted by a name (see fig. 19, below). Then it is evident that two names mean a certain arrangement consisting of two cubes in definite places with regard to one another—three names denote three cubes, and so on. And I would ask the reader not to mind taking a little trouble at this point, and to look at the diagram for a little while. If there is anything about which we can form perfectly clear ideas, it is a little heap of cubes. And if the reader will simply look at them for a little space of time, he will realize clearly every word of what I have to say; for I am going to talk about nothing else than this little block of cubes.

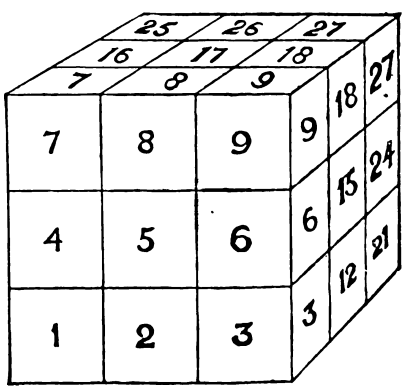


FIGURE 19. A block consisting of 27 cubes.

Thus, looking at the cube with the number 1 upon it, this numeral will serve for the name of the cube, and similarly the number written on every cube will serve for its name. So if I say cubes 1 and 2, I mean

[11] A cubic yard contains 46,656 one-inch cubes. It seems evident that the way in which Hinton learned different names for each of these cubes was to first learn a one-word name for each cube in a 6 x 6 x 6 block of six-inch cubes. This would be 216 names, and Hinton actually gives these names as Appendix B to *A New Era of Thought*. The names are a more or less random collection of Latin nouns, e.g., Fons, Plectrum, Vulnus, Arena, Mensa, Terminus, Uva, Coma, and so on. Each of the six-inch cubes consists of a block of 216 one-inch cubes. Thus a name such as Fons Uva would be used to mean a one-inch cube which was located by first finding the six-inch cube in the Fons position, and then finding in that cube the one-inch cube in the Uva position. [R.v.B.R.]

the two which lie next to each other, as shown in the diagram; and the numbers 1, 4, 7, denote three cubes standing above each other. If I say cubes 1 and 10, I mean the first cube and one behind it hidden by it in the diagram.

Now this is the bit of knowledge on which I propose to demonstrate the process of casting out the self. It is not a high form of knowledge, but it is a bit of knowledge with as little ignorance in it as we can have; and just as it is permitted a worm or reptile to live and breathe, so on this rudimentary form of knowledge we may be able to demonstrate the functions of the mind.

And first of all, when I had learnt the cubes, I found that I invariably associated some with the idea of being above others. When two names were said, I had the idea of a direction of up and down. But with regard to the cubes themselves, there was no absolute direction of up or down. I only conceive of an up and down in virtue of being on the earth's surface, and because of the frequent experience of weight. Now this condition affecting myself I found was present in my knowledge of the cubes. When certain of the names were said, I conceived of a figure having an upper part and a lower part. Now, considered as a set of cubes related to one another and not to me, the block had nothing to do with up and down. As long ago as Ptolemy, men have known that there is no such thing as an absolute up and an absolute down. And yet I found that in my knowledge of the set of cubes there was firmly embedded this absolute up and this absolute down. Here, then, was an element arising from the particular conditions under which I was placed, and the next step after recognizing it was to cast it out. This was easily done. The block had to be turned upside down and learnt over again with the cubes all in their new positions. It was, I found, quite necessary to learn them all over again, for, if not, I found that I simply went over them mentally the way first learnt, and then about any particular one made the alteration required, by a rule. Unless they were learnt all over again the new knowledge of them was a mere external and simulated affair, and the up and down would be cast out in name, not in reality. It would be a curious kind of knowing, indeed, if one had to reflect what one knew and then, to get the facts, say the opposite.

It may seem as if, when the cubes were known in an upright position, they would be easily imagined in an inverted position. But practice shows that this is very far from being the case. It requires

considerable mental effort to determine the alterations in position, and to get an immediate knowledge requires a considerable time.

It may seem as if it were a dubious way of getting rid of gravity, or up and down, just to reverse the action of it.

But this way is the only way, for we, I have found, cannot conceive it away; we have to conceive it acting every way, then, affecting each view impartially, it affects none more than another, and is practically eliminated.

The cube had not only to be turned upside down, but also laid on each of its sides and then learnt. There were a considerable number of positions, twenty-four in number, which had to be brought close to the mind, so that the lie of each cube, relative to its neighbors and the whole block, was a matter of immediate apprehension in each of the positions.

If a single cube be taken and moved about, it will be found that there are twenty-four positions in which it can be put by turning it, keeping one point fixed, and letting each turning be a twist of a right angle. The whole block had to be turned into each of these positions and learnt in each.

Thus the block of cubes seemed to be thoroughly known.

At any rate, up and down was cast out. And we can now attach a definite meaning to the expression "casting out the self." One's own particular relation to any object, or group of objects, presents itself to us as qualities affecting those objects—influencing our feeling with regard to them, and making us perceive something in them which is not really there.

Thus up and down is not really in the set of cubes.

Now these qualities or apparent facts of the objects can be got rid of one at a time. To cast out the self is to get rid of them altogether.

As soon as I had got rid of Up and Down out of the set of cubes I was struck by a curious fact.

If in building up the block of cubes one *goes* to the left instead of to the right, keeping all other directions the same, a new cube is built up having a curious relation to the old cube. It is like the looking-glass image of the old cube. Every cube in the new block corresponds to every cube in the old block, but in the new figure it is as much to the left as before it was to the right. And any set of names in the block so put up gives a shape which is like the shape denoted by the same set of names in the old block, but which cannot be made to coincide with it,

however turned about. It is the looking-glass image of the old shape. The one block was just like the other block, except that right was changed into left. Now, was it necessary to cast out right and left as had been done with up and down? or was right and left, as giving distinctions in the block and in shapes formed of cubes, to remain? It seemed as if right and left belonged more to me than to the set of cubes. And yet the right-handed set of cubes could not be made by

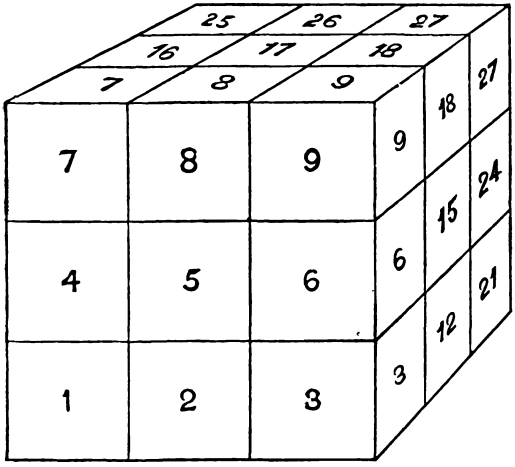


FIGURE 20. Diagram is a block.

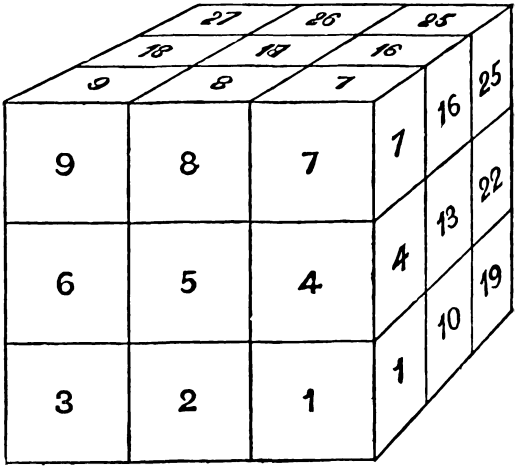


FIGURE 21. Diagram is the image block of figure 20.

moving about to coincide with the left-handed set of cubes. And this power of coincidence was the test which had convinced me of the self nature of "Up and Down."

If, putting up the block of cubes, we go to the left instead of the right, but in all other respects build up in the same way, we obtain a block which has a curious relation to the first block.

The ordinary block is shown over again in figure 20. Figure 21 is the new block. The new block is like a looking-glass image of the old block. It is just the same, but that left and right is reversed.

Also, if we take selections of blocks we get figures which are just reversed. Thus 1, 4, 7, 8, in Block 20 means a figure turned to the right; in Block 21 a figure turned to the left.

Again, consider the two figures formed by selecting the cubes 1, 4, 7, 8, 17, from figures 20 and 21 respectively. We get two figures which are just like one another as arrangements, but which we cannot turn into one another by twisting.

Considered as arrangements in themselves, these figures and these blocks seem to be identical, for the relationships of cube to cube which are present in the one are all present in the other. But considered as shapes they are not identical. For they will not coincide.

There is another manner of regarding the matter which may help to bring out the point at issue.

If we suppose that we are putting up the cubes in one room while another person is putting up cubes in an adjoining room; if we can tell him what we are doing, using the words right and left, he will be able to put up a block exactly like ours. But if we do not allow ourselves to use the words right and left, but speak to the other person as if he were simply an intelligence without having the same kind of bodily organization as ourselves, we should find that, supposing he could put up the block of cubes, it would be a mere matter of chance whether he had put up the block as we had put it, or whether he had put it up in an image way. And the same with regard to any shape. We could tell him that the cubes should be put together, and we could tell him the relationship which they should have with regard to one another; but the figure he put up would just as likely be an image of our shape as not.

And we could go on for ever building more and more complicated shapes and telling him to do the same, and no hitch or difficulty would come. But at the end all his shapes might be ours just reversed, as if seen in a mirror.

And if, having put up the block, we colored the sides of the cube we used as the fundamental cube, and told him how we had colored it: if he colored his and brought it to us, and we compared them, his would just as likely be the image of our cube, and not able to be turned into it. So that although, as arrangements, the structures we had put up were alike, still neither of us could use the other's fundamental cube; and if we exchanged the fundamental cubes there would be an inconsistency in each of our arrangements.

Now, are these blocks of cubes really the same? Are 20 and 21 really the same in themselves, as all relationships in the one are to be found in the other? If so, the feeling on my part that they are different, and the inconceivability of their coinciding, must be due to some self-element which is mixed up with my apprehension of the cube.

Now, if "right and left" was a self-element, it could be cast out; if it was a permanent distinction in the cubes themselves, it could not be cast out. The thing to do was evidently to try. The method was to learn the cubes over again, in a set of new positions. For every one of the ways in which they were learnt before, there was an inverted or pulled through way to be learnt.

While I was engaged in this attempt another inquiry suddenly coincided with this, and explained it all.

Much has been said about the fourth dimension of space and the inconceivability of it to us. Now, if there are beings who live in a four-dimensional world, they must feel as habituated to it as we do to ours, and the conceptions which seem so impossible to us must be every-day matters to them. It would be impossible for us to try to enter at once into the serious thoughts of these denizens of higher space. But amongst them there would probably be some with whose occupations we might become familiar, and with whose ideas we might gain some acquaintance. Amongst these beings there must be children, and just as children on the earth gain their familiarity with space by means of bricks and blocks and toys, so these higher children must have their own simple objects wherewith they grow into familiarity with their complex world.

Now it is easy to make a set of simple objects such as these higher children would use. And it seemed a practical thing to do with regard to the conceivability or inconceivability of the fourth dimension to give the matter a fair trial, by going through those processes and those experiences which must be gone through by the beings in higher space to gain their acquaintance with it.

When I say that it is easy to make a set of objects, such as the higher children use, I do not mean to say that they can be made completely in every part at once. But we can make the ends and sides of them, and we can look at the ends and sides of them as they appear to us in space, and we can make up exactly what sides come into space when the simple objects are twisted and moved.

Just as a being living on a plane could tell about all the faces and edges of a cube or other simple solid figure by looking at what he could see when the cube was laid on his plane, and when it was twisted and laid down again; so we can tell all about the sides, faces, and edges of a higher solid.

And the project seems less uninviting if we reflect on how complicated a matter the formation of our own conceptions of a solid are. What a lot of faces and edges a cube has! And, moreover, it must be remembered that we never touch or see a solid; we only see the surface and touch the surface. If we cut away the surface that we first saw or touched, we come on another surface, and so on.

Now, of course, the surfaces of a solid are given to us by nature in their right connection and relation. Each of the edges of the cube, for instance, can be noticed and remarked without any difficulty, and they are all on the same bit of space, to be looked at one at the same time as another.

But the sides, faces, and edges of a higher solid cannot be in our space all at once. They must come separately, be looked at one by one.

Thus a being in a plane could not see the lower side and the front of a cube at once. He would first have to look at the lower side as the cube rested on his plane, then if the cube were turned over he would see the front, and the lower side would be gone. If he got the set of right appearances which a cube would present to him when, turning about in a systematic way, it came at intervals into his plane, and if, moreover, he fixed his mind on these appearances, he might at last, if it was in him, rise to the conception of a cube as we know it.

Now, the parts by which a higher solid comes into our space are solids, and what we have to form is a set of solids coming and going in a systematic way, as the higher figure is moved about in a systematic way.

This afforded a welcome exercise, for conceiving the solid shapes, and how they went and came, increased my familiarity with the set of cubes.

Moreover, in trying to get the piece of ignorance—the necessary real cube—as small as possible, I had got the block which I knew to a somewhat fine state of division, and could, by picking out a particular set of cubes from the whole number, obtain a mental model of any shape I wanted. The whole block of cubes formed a kind of solid paper in which one could mentally put down any solid shape one wanted. And just as it is a great convenience to have a piece of paper for drawing figures one wants to think about, so it was a great convenience to have this solid paper.

The subject, however, abounds in abysses for stupidity to fall into, and I had to clamber out of each of them; so it took me several years before I got quite on the right tack. Then it was easy enough: any one in a few weeks could learn to conceive four-dimensional figures. Not only is it easy, but there are abundant traces that we do it continually without being aware of it. I am sure if the loveliness of the work while one is doing it, and the simplicity and self-evident nature of the results when obtained, were generally known, it would be a favorite amusement.

Now one of the first things that presented itself to my attention when I began to move the four-dimensional figures about was a fact which bore curious reference to my difficulty about the fundamental cube. If the reader remembers, it seemed to me as if the cube out of which the whole block of known cubes was built ought to be able to be inverted. That is to say, it seemed to me that there was a self-element present in my knowledge of the cubes. But in order to cast out that self-element the fundamental cube which lay at the basis of the whole block would have to be able to be inverted, or pulled through.

Now I found that when I took a four-dimensional figure which came into space by a cube—that is, a figure which rested on space by a cube, or one of whose sides was a cube—when I took a figure of that sort up in the fourth dimension and twisted it round and brought it down again, this cube would sometimes be inverted or pulled through—although I had done nothing to it, but had simply twisted the whole figure round without disturbing the arrangement of its parts.

Thus evidently to a higher child it would be no more difficult to invert or pull through a cube or a figure than it would be to me to twist one round.

Hence it was obvious that right and left was really a self-element in

my block of cubes. A being in our space was under a certain limitation, and that limitation made me feel as if a right-handed arrangement was different from a left-handed arrangement.

A being who was not limited as I was would see that they were one and the same. Hence, in knowing the set of blocks it was necessary to cast out "right and left," and the names had to be learnt over again in new positions.

Thus it is evident that there are three expressions which may be considered in reference to a knowledge of a block of cubes as almost identical: "Casting out the self"—"Seeing as a higher child"—and thirdly, "Acquiring an intuitive knowledge of four-dimensional space."

Thus, taking the simplest and most obvious facts—the arrangement of a few cubes—we found that there was a known part and an unknown part; the known part corresponding to our act of putting, the unknown part the cube which, of some size or another, must be taken as given in the external world. Then there was obviously a self-element present in the Up and Down felt as in the cubes. This being removed, Right and Left had also to go. So, to get the knowledge of this simple set of objects clear of self-elements, two universe transforming thoughts have to be used; and when these thoughts are thus incorporated the cubes become different.

MANY DIMENSIONS

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In connection with the subject of higher space there is a remark which is sometimes made, a question which is put—"If there are four dimensions, then there may be five and six, and so on up to any number?"

This question is one, I own, which it would never have occurred to me to ask. Still, it often happens that a line of thought which is most foreign and unattractive does repay investigation. And so let us follow the ready algebraist, to whom it is as easy to write down five as four, and n as five. Let us see what it is reasonable to think on the subject.

If we take four-dimensional shapes and examine them, we find that there is in them a peculiarity of the same kind which led us to be sure of the reality of a four-dimensional existence from the inspection of these dimensional shapes. In four dimensions we can have two figures which are precisely similar in all their parts, and which yet will not move so that one shall occupy the place of the other.

And the same observation can be made with regard to five-dimensional figures.

Hence it would seem that there is an indication of a higher and higher reality. And if we suppose that the same fact of absolute similarity, without the possibility of superposition, were found again and again, then we should be compelled to recognize the existence of higher and still higher space, and we should have to admit the existence of an indefinite number of dimensions.

But let us turn away from this direct inquiry. Let us ask what the phrase "an infinite number of dimensions" denotes.

The question reminds me so forcibly of an Eastern story that I must digress for a moment.

For it is said that once, in the cool of the morning, beneath the spreading branches of a great palm the master stood. And round him were gathered three or four with whom he spent the hours of his quiet life.

And not for long had they gathered together.

One was a warrior, and long ago he had come to the master, asking him what he should do, and had received for answer—"Go back and serve your commander. The day will come when you will have fulfilled your life, and the voice within you will speak clearly."

And the soldier had returned to the life of camps and marches and combats, till at length, at the close of a hard-fought day, he threw down his weapons, and passing through the enemy's land, came to where the master taught.

And his comrades, seeking long for their leader, at last buried with honor a corpse unrecognizable for wounds.

He now sat on a bare stone listening. Beside him stood a younger man. He had been a merchant, travelling over the whole earth in search of gain, and in restlessness of curiosity. And when in wonder he had begged the master what he should do, he had been told—"Wander over the earth, and visit every part; when thy eagerness for change is satisfied, an inward voice will lead thee."

And he had travelled far, till, even, in the course of his wanderings, he had come to the most distant lands, and gained great riches by what he bought and sold.

But when his stores were full, and his possessions had increased beyond his dreams, he left them all, and, seeking that hillside, lived obediently to the master's words.

Half lying upon the ground was one whose countenance hardly bespoke him a fitting companion for the others. And, indeed, he had been that one, whose life had afforded the master the most interest of all of them.

For he had not, like the others, been immersed in an active and adventurous life, but had been a slave to the wants of his own body. And seeing amidst his vices that the master had words for others, he had besought him to tell him too what to do.

And the master had told him first one thing and then another, but always he fell back, unable to withdraw himself, even for a short while, from his bodily cravings, but, gratifying them with drink and sloth, he passed his days in brutishness.

Then at length the master, hailing him as a friend, had said to him—"I will not seek to withdraw you any longer, for is not your body like the rain-clouds, and the sky a part of the changing show that hangs before our faces? Gaze, therefore, earnestly on your body, attend to it the more intently, for this is your vocation; and when you see the flimsy veil it is, come to me."

And this man had sat for ten years contemplating the middle portion of his body, till his frame had grown so cramped that he could not rise. At last he had bidden his fellows carry him to the master; and now he too listened to the words that fell on welcome ears.

And many days they had spoken together, and retiring each to his hut of reeds at nightfall, had pondered over the master's words. And on each of them had come a change.

Into the soldier's face, hard and stern set, had come the dawn of gentleness. The quick, observant gaze of the traveller now at times changed almost into such an expression as one would wear who looks at the wide fields that lie above the countries of the earth. And in the dull, inexpressive countenance of him who had sat absorbed in the contemplation of his body, had come the kindling light of intelligence.

And on this day the master opened his lips, and began to instruct them about the universe.

He told them much that made them wonder. He told them of the mysterious currents of life that passed away from the bodies and frames which they could see, and that, spreading into the minutest particles of the earth, collected again, and eddying back through seed and leaf and fruit, participated anew with the soul, which also in its turn had gone through many vicissitudes, in that mingling ground of various principles which we call a human life.

And seeing their wonder and interest, and feeling that they were desirous to know, and since, moreover, he saw no harm in gratifying their wish, he began to explain to them the deepest facts of their physical being. And talking of the universe, which contained all that they saw and knew, from the beneficent stars to the humblest blade of grass, he said—"The world rests upon an elephant." And then he paused.

The warrior did not speak. He who had been absorbed in the contemplation of his body did not open his lips—or if he had it would not have mattered; for with the instinctive and right attitude of the half-cultured mind to the proximate object which is the last to come before its intelligence, he would have said if he had spoken, "I worship the elephant;" and the master would have greeted this remark with a kindly smile, and proceeded with his discourse.

But just as he was about to take up the thread of his speech, there came from the traveller, who had been listening eagerly, a hurried question.

For, alas, in his wanderings, this one had traversed the greater part of the globe, and in the course of them had come to the West, where

even at this early period a habit of mind reigned, very unlike that which characterized the calm, deep, contemplative souls of the East.

Moved by this restless and questioning spirit, he cried out—"And on what does the elephant rest?"

"Upon a tortoise," the holy man replied. And had he not been beyond all human passions, his tone would have been one of mockery.

He taught them no more. Why should he tell them of these things? Was it not better rather to dwell in the daily perfectionment of brotherly love, and in the ministering offices of devoted lives?

And yet one cannot help wishing that unlucky question had not been put. If only the unfortunate disciple had but said, "Let us investigate the elephant," or, better still, had said nothing—what should we not have known now!

And if then such a question sealed the fount of sacred wisdom at that remote epoch, what must not the effect of our modern mind be?

For now such a disciple would not simply ask, "Upon what does the elephant rest?" but he would have glibly asked, all in one breath—"Upon what does the elephant rest, and upon what does the support of the elephant rest, and on what the support of that? and so on, ad infinitum; do tell me."

And so too, even on the rivulet from the fount of wisdom that trickles sparingly through our own minds, is there not a checking effect coming from this mental attitude of ever asking what is behind and behind and behind, seeking formal causes always, instead of living apprehension of the proximate?

Indeed, that question was a misfortune if the possession of fact knowledge is a boon. For what could have been a more apt description of this all-supporting elastic solid ether than the broad arching back of the largest animal known on earth—the created being that could bear the most, and of all not-human creatures, the most intelligent and responsive?

The master knew how all the worlds were held together—and how much more!

And, indeed, does not this feeling come upon us strongly with regard to those of the Eastern world, with whom we have the privilege of talking?

For my own part, however much I have learnt in the intervals of my speaking with them, there they still hover on the weather-bow of my knowledge—they, or those from whom they learn, are in the possession of knowledge of which all my powers are but secondary instances or applications.

What it is I know not, nor do they ever approach to tell me. Yet with them I feel an inward sympathy, for I too, as they, have an inward communion and delight, with a source lying above all points and turns and proofs—an inward companion, whose presence in my mind for one half-hour is worth more to me than all the cosmogonies that I have ever read of, and of which all the thoughts I have ever thought are but minutest fragments, mixed up with ignorance and error. What their secret is I know not, mine is humble enough—the inward apprehension of space.

And I have often thought, travelling by railway, when between the dark underground stations the lads and errand boys bend over the scraps of badly printed paper, reading fearful tales—I have often thought how much better it would be if they were doing that which I may call "communing with space." 'Twould be of infinite delight, romance, and interest; far more than are those creased tawdry papers, with no form in themselves or in their contents.

And yet, looking at the same printed papers, being curious, and looking deeper and deeper into them with a microscope, I have seen that in splodgy ink stroke and dull fibrous texture, each part was definite, exact, absolutely so far and no farther, punctiliously correct; and deeper and deeper lying a wealth of form, a rich variety and amplitude of shapes, that in a moment leapt higher than my wildest dreams could conceive.

And then I have felt as one would do if the dark waters of a manufacturing town were suddenly to part, and from them, in them, and through them, were to uprise Aphrodite, radiant, undimmed, flashing her way to the blue beyond the smoke; for there, in these crabbed marks and crumpled paper, there, if you but look, is space herself, in all her infinite determinations of form.

Thus the reverent and true attitude is, not to put formal questions, but to press that which we know of into living contact with our minds.

And so the next step, when we would pass beyond the knowledge of the things about us in the world, is to acquire a sense and living apprehension of four-dimensional space.

But the question does come to many minds. "What lies beyond?" And, although our knowledge is not ripe enough to answer this question, still, hurrying on before, we may ask—not what does lie beyond, but what is it natural for us in our present state of knowledge to think about the many dimensions of space?

Let us drop for a moment into the most common sense mode of looking at it. Why do we think of space at all? To explain what goes on. If

everything followed uniformly, we should not need to think of three, or even two dimensions—one would do. But problems come up, practical problems, which need to be reconciled. Things get "behind" one another, are hidden, and disappear. So we find that one variable will not suffice. If we were in a line looking at only one thing, its gradual changes of distance from us would be all our experience. We should not call this "distance"; it would be the one fact of our experience; and if we treated it mathematically, we should express it as *the variation of one variable.* So we may consider as identical, one-dimensional space, and the variation of one variable. Now plane space requires two variables. May not plane space then be defined as our knowledge of the variation of two variables? The being in plane space requires two variables to account for his experience. He lives, we say, in a space of two dimensions.

Now why should we not identify these, and say that that which he calls space is the organized mass of knowledge of the relations of two variables that has grown up in his mind?

We talk of distance and size as if each were something known in itself. But suppose a percipient soul subjected to a series of changes depending on two independent causes, which always operated together, and which were each of them continuous in their increase and diminution, would not this percipient soul form an idea of space of two dimensions? Would he not say that he lived in a space of two dimensions? His apprehension of the number of variables by which he was able to account for his experience would project itself into a feeling of being in space; and the kind of space would depend on the number of variables he habitually worked with.

Now we have become habituated to use, for practical thought, three variables; these explain the greater part of our daily life. Is that which we call space simply the organized knowledge of the relations of these variables? Without pledging ourselves to this view, let us adopt it and note its consequences.

Then it is evident that as we come into the presence of more and more independent causes—I mean, as we find that these are in nature working independently of one or more in number than three—we shall have to study the general aspect of events which turn up from the combinations in varying intensity of these four or more principles, or causes of our sensation. Then we shall get a mental organization capable of dealing readily and rapidly with the combinations of these causes. And this mental organization will be indicated in our con-

sciousness by the feeling of being in four-dimensional (or more-dimensional) space.

It seems strange to talk of there being three independent causes, or of some such limited number, for in the events that happen around us we see a vast variety of causes. There is the tendency to fall, there is the motion of the wind, there are the actions of human beings, each of them producing effects, and besides these many other causes.

But if we look at them, we find that they are not all independent one of the other, but may be different forms of the same cause.

Indeed, if we suppose that we live in three-dimensional space, and that every change and occurrence is the result of the movements of the small particles of matter, there would ultimately be only three independent causes—the three independent movements, namely, which a particle could go through.

Thus it would appear that, since no one would deny that there are an infinite number of perfectly independent causes in nature, the formation of a sense of higher and higher kinds of space was simply necessary as, our knowledge becoming deeper, we came into contact with more and more of these causes.

It might be said that these causes might be very diverse from each other; one might be apprehended as love, another as color, another as distance. But this view is hardly tenable, for to apprehend a cause it must be congruent with the others which we already apprehend. If it is known at all it must work uniformly in with the rest of our experience. No doubt there are an infinite number of causes, which give that richness to experience of which the intellect can take hold only by a small part. But when the intellect does take hold of a part, it takes hold of it by seeing how it comes in, modifying each of the already existing possibilities and producing a new variety, out of which the actual experience is a selection. Thus, if a being having an experience derived from two causes, and so living in a space of two dimensions, were to be affected by a third cause, he would first of all find that there were many things which he would say could not be explained by space relations. Then he would gradually arrive at the idea of a three-dimensional space. Space being due then not to anything in the nature of the causes themselves, but to the number of them.

Then, to us, when mentally we come into the comprehension of any new independent cause, we must acquire the sense of a new dimension, and the question of space and space relation is altogether inde-

pendent of the nature of these causes—the real and systematic apprehension of them necessitating an enlargement of our sense of space. Now the unknown comes to us generally in the properties of the minute particles of matter which make the different "kinds." Hence, as we study matter closer and closer we shall find that we need more and more dimensions.[12] And the molecular forces in one kind of space will be the physical forces of the next higher.

That is to say, when in our space we have explained all that we can explain by the supposition of particles moving in our space, we shall find that there is a residuum, and this residuum will be explained by the four-dimensional movement of the minutest particles. The large movements are simply movements in three-dimensional space, but to explain the residual phenomenon a higher kind of space will be requisite.

Still, this all seems to me a barren view, and I am convinced that it is far truer to think of space, as indeed we can hardly help doing, as a beneficent being, supporting us all, looking at us in every lovely leafy bough, and bending towards us in the forms of those we know.

And, moreover, there is one very valid objection to the conclusion that we have explained anything, or made any step by using the word "variable."

It will be found that such a notion as a continuously varying quantity is a mere verbal expression. All that we can conceive or understand are definite steps, definite units. We can conceive a great many definite magnitudes, but not continuous magnitude. The idea of continuity is one which we use and apply; but to think men have explained anything by speaking of continuous variables, is really to lose ourselves in words.

But, although we dismiss the previous supposition, still we see that, even if it were true, the practical thing to do is to acquire the sense of a higher dimensional space.

And, indeed, what a field is here! Take a single example. The idea of magnitude is one dimensional—simply adding and adding on in a straight line.

The idea of rotation, or twisting, in its very nature involves the idea of two dimensions—for it is the passage from one dimension to another—it is an idea which, in its essence, has two dimensions.

[12] It is in fact now customary in quantum mechanics to treat particles as structures in infinite-dimensional Hilbert spaces. [R.v.B.R.]

If we think of a twist, it is the change from one direction to another. It cannot be thought without the two directions being present to the mind—the direction from which and the direction to which the change takes place.

In our space we have nothing more than this rotation. If a ball is twisting, and a blow is given to it, which tends to set up a twist in a new direction, the old twist and the new one combine together into a single twist about a new axis.

But in four-dimensional space there is such a thing as a twist of a twist—a rotation of a rotation—bearing to a simple rotation the same relation that an area does to a line. Perfectly independent rotations may exist in a four-dimensional body.[13]

And again, evidently if there is an idea which in its essence involves two dimensions, may there not be an idea which, of its very nature, includes three dimensions?

What that idea is, we do not know now; but some time, when the knowledge of space is more highly developed, that idea will become as familiar to us as the idea of a twist is now.

And, indeed, space is wonderful. We all know that space is infinite in magnitude—stretching on endlessly.

And when we look quietly at space, she shows us at once that she has infinite dimensions.

And yet, both in magnitudes and dimensions there is something artificial.

To measure, we must begin somewhere, but in space there is no "somewhere" marked out for us to begin at. This measuring is something, after all, foreign to space, introduced by us for our convenience.

And as to dimensions, in order to enumerate and realize the different dimensions, we must fix on a particular line to begin with, and then draw other lines at right angles to this one.

But the first straight line we take can be drawn in an infinite number of directions. Why should we take any particular one?

If we take any particular line, we do something arbitrary, of our own will and decision, not given to us naturally by space.

No wonder then that if we take such a course we are committed to an endless task.

[13] This is explained in the paper "The Recognition of the Fourth Dimension," pp. 142–162 of this edition. [R.v.B.R.]

We feel that all these efforts, necessary as they are to us to apprehend space, have nothing to do with space herself. We introduce something of our own, and are lost in the complexities which this brings about.

May we not compare ourselves to those Egyptian priests who, worshipping a veiled divinity, laid on her and wrapped her about ever with richer garments, and decked her with fairer raiment.

So we wrap round space our garments of magnitude and vesture of many dimensions.

Till suddenly, to us as to them, as with a forward tilt of the shoulders, the divinity moves, and the raiment and robes fall to the ground, leaving the divinity herself, revealed, but invisible; not seen, but somehow felt to be there.

And these are not empty words. For the one space which is not this form or that form, not this figure or that figure, but which is to be known by us whenever we regard the least details of the visible world—this space can be apprehended. It is not the shapes and things we know, but space is to be apprehended in them.

The true apprehension and worship of space lies in the grasp of varied details of shape and form, all of which, in their exactness and precision, pass into the one great apprehension.

And we must remember that this apprehension does not lie in the talking about it. It cannot be conveyed in description.

We must beware of the attitude of standing open-mouthed just because there is so much mechanics which we do not understand. Surely there is no mechanics which we do not understand, but geometry and mathematics only spring up there where we, in our imperfect way, introducing our own limitations, tend towards the knowledge of inscrutable nature.

If we want to pass on and on till magnitude and dimensions disappear, is it not done for us already? That reality, where magnitudes and dimensions are not, is simple and about us. For passing thus on and on we lose ourselves, but find the clue again in the apprehension of the simplest acts of human goodness, in the most rudimentary recognition of another human soul wherein is neither magnitude nor dimension, and yet all is real.

The answer to this is twofold. In order to live, self knowledge is necessary. That knowledge of self which is distinctly a matter of ethical inquiry, is altogether foreign to these pages.

But there is a no less important branch of self knowledge which

seems altogether like a research into the external world. In this we pass into a closer and closer contemplation of material things and relations, till suddenly we find that what we thought was certain and solid thought is really a vast and over-arching crust, whose limitlessness to us was but our conformity to its limits—a shell out of which and beyond which we may at any time pass.

But if we do so pass, we do not leave behind us the idea of matter. All that we thus attain is a different material conception of ourselves.

In ancient times there was no well-defined line between physics and metaphysics. And our present physical notions are derived from amongst the mass of metaphysical notions. Metaphysics is so uncertain, because when any one of its doctrines becomes certain, it takes a place in physics.

And the exploration of the facts of higher space is the practical execution of the great vision of Kant. He turned thought in an entirely new direction. And where he turned, all seemed blank—all positive assertions fell away, as he looked into the blackness of pure thought.

But out of this absence can come any amount of physical knowledge. It is like an invisible stuff out of which visible garments can be woven.

But, indeed, many would say: What is the use of these speculations?

Does not the contemplation of space leave the mind cold, the heart untouched? Not altogether.

Is not our life very much a matter of fact, concerned with events? All our feelings are bound up with things which we do or suffer.

And thus a right conception of the possibilities of action in our world, and in a higher world, must have some influence on ourselves.

Then also there is a path through which we can pass, leading from the most complete materialism to something very different from the first form in which it presents itself.

Any one, who will try, can find that, by passing deeper and deeper into absolute observation of matter, and familiarity with it, that which he first felt as real passes away—though still there, it passes away, and becomes but the outward sign of realities infinitely greater.

Thus there springs before the mind an idealism which is more real than matter; a glimpse of a higher world, which is no abstraction, or fancy, or thought, but of which our realities are the appearances.

And with this there comes overpoweringly upon the mind of one, who thinks on higher space, the certainty that all we think, or do, or imagine, lies open.

In that large world our secrets lie as clear as the secrets of a plane being lie to an eye above the plane. For howsoever closely a being living on a plane may hide from his fellows, he has nothing secret from an eye that gazes down upon his plane.

The very idea that he can put forward to such a one any false pretenses, is absurd.

And so we lie palpable, open. There is no such thing as secrecy.

And as I have said before, the difference between the moral life and the animal life, in a world of any dimensions, lies in this—that the animal life consists of actions which are those natural to the possibilities of space of that world; the moral life (viewed as exhibited in physical arrangements) lies in the striving, by modification and restraint of the natural actions, towards those actions and modes of existence which are natural in a higher space world.

It has been shown how plane beings could only pass each other by courtesy and mutual forbearance. And the great effort wherein the higher spirit most plainly shows itself, apart from convenience, or profit, or any obvious physical good, is in one very simple and obvious tendency towards a higher-dimensional existence. For, as to a higher space being no secrets of ours are hidden—nothing is unknown, so, in making towards one another our limited lives open and manifest, we treat each other in the service of truth, as if we were each members of that higher world.

It is often said and felt, that all our actions do in the course of time impress their effect on the world. Nothing is lost. And if we, being limited, know that this is so, how much the more apparent is it when we realize our higher being. We know that, as animal frames moving and acting in the world, the effect of every movement passes on and on.

And with this effort corruption and evil fall. Space is so large that no interior can be hidden from the vivifying breath of the universe; no part can be cut off, however foul, from direct contact with the purifying winds which traverse space higher than itself.

As conscious minds, we realize the oneness of past and future in our open communication one with another. We attain a mental consciousness of the higher fact. Whether we represent it to ourselves as a day wherein all that ever has been done will be told, or as an omnipresent and all-knowing mind, it is the same.

Truth is nothing but an aspiration to our higher being. And the first sign of love towards individuals, as towards the world—as distin-

guished from the easy and yielding good nature which always tries to please that which is nearest at the moment—is veracity. This is the secret of the mysterious effect of science on our emotions—the simple description of fact, apart from our own conditions and prejudices. And also in the material world around us, this is the secret of the beauty of the crystal and of still water. For in them the near and the far are brought together; in their translucency they give an emblem of the one vision wherein a higher being grasps every part of the solid matter, of which we can only see outside and surface.

The acceptance of the rule of the great master of empiric religion, Comte—"Live openly"—is really to imitate in our world, and make ourselves conscious of our true existence in a higher world.

There are two sides of religion—the inductive and the deductive. To the realm of deduction belongs theology, with its central assertion and its manifold consequences. But inductive religion consists in grasping, amidst the puzzling facts of life, those greater existences in which the individual organizations are bound up, and which they serve, passing, as in every science, from the details to the whole. And the connecting link between materialism and the conduct of life, lies in the doctrine of the limited nature of our present space perceptions. For, with the elevation of our notion of space to its true place, the antagonism between our present materialistic and our present idealistic views of life falls away.

AN UNFINISHED COMMUNICATION

[This selection contains excerpts of "An Unfinished Communication" from *Scientific Romances,* Vol. 2 (1885). It includes material from all chapters.]

Mr. Smith, Unlearner

I was traversing a quarter of New York in which the streets, winding and squalid, with low buildings on either hand, resemble the alleys of an old world city. Turning a corner, I was in the most dejected of all that I had passed. Some vigorous, successful effort no doubt had its origin here, but the street was more like the last halting-place but one in a weary and dispirited struggle.

Yet even here were signs of that feverish activity which makes even the most squalid streets of our land different from those of the old world. Little stores wedged themselves in the basements; narrow, strip-like eating-houses thrust themselves into impossibly small spaces. Here and there a bell-pull wrenched off betokened the common lodging-house, while scattered over all the grimy windows and doors were the notices of decaying trades, the signs of struggling hand industries, the advertisements of professors in the last stage of indigence. Boot-makers, flower-makers, a dancing mistress, piano lessons, shorthand in five weeks, all these I read. Jetson Street seemed a favorite camping-ground for the cheap lesson-giver. Wearied with these sordid details, I passed rapidly on, buried in my thoughts.

But even in the least attentive mood an unusual word, a misspelling, an incongruity of any kind has the power to attract the eye and cause it to send a signal to the brain. There was a house opposite which, by its carefully patched and preserved paint, its unbroken railings, seemed to suggest that it had struggled painfully to preserve appearances, and only succumbed finally because after all it was in Jetson Street. On the door was the unusual notice that took my eye. A plain drab board had the words—

MR. SMITH,
UNLEARNER.

Of all the misdirected efforts towards earning a livelihood this, I thought, is the most futile, and I paused to smile at the foolishness that put it up. What weary, dull-eyed failure was it who, unable to succeed in any pursuit, advertised himself as willing to impart his incapacity to others? But as I stopped there somehow came into my mind the idea, that genuine services such as this foolish creature pretended to offer might not be unacceptable to myself. How pleasant it would be to let pass away some of that verbiage I learnt at school—learnt because teachers must live, I suppose. The apeing and prolonged caw called grammar, the cackling of the human hen over the egg of language—I should like to unlearn grammar. The sense came over me, faintly at first, but gathering strength, of how much I should owe to any man who would rid me of what I learned at college—that plastering over of the face of nature, that series of tricks and devices whereby they teach a man knowing nothing of reality to talk of it as if he did. There passed before my mind that pallid series of ghosts, ghosts of what had once been some man's living, practical work, the books by which professors—because they must live, I suppose—keep younger men from life and work.

A gleam of hope came over me that I might forget my philosophy lectures and the teachings of that bespectacled Doctor of all the sciences, who always turned the handle the wrong way, while he told us the principles by which things go. The line at infinity, it would be nice to forget that; and the unconscious will—the principle of being and not being too, which, not much in itself, yet, like an active commercial traveller, makes business at both ends.

It would be pleasant too to forget the Darwinian theory, which tells me things are as they are because they are not something else; and astronomy, which kicks the globe into companionless cold space; and physics, which tells us we are but the result of multitudes of moving particles. If all these were to sink and disappear from me, then perhaps I should be face to face with something not a specter, not an instance and example of a phase, a formula, a barren set of words.

The letters stared me in the face unmistakably, for I had approached close to the house. When seen near, its apparent superiority to its surroundings vanished. But the inscription on the board was clear, emphatic, and, as paint was estimated in that street, of not such very ancient date.

Perhaps here, in this obscure corner, is some neglected philosopher, who, like Socrates, can teach a willing listener that he does not know.

Perhaps at his words that hollow crust will crumble away, that is each man's idea of himself and his fellows, letting the man himself be known.

Could he really teach me not to know—to be as though I had not known—would not that be to forget? To pass out of those shades of vast and poisonous thoughts coeval with the race? Could he wipe out those foul ideas that pollute man's strength and woman's beauty—can he make the mind as though they had never been? Can he take away those thoughts that cast their withering shade over earth's fair flowers and turn the man to brute? Can he put an end to divided endeavors and self-contemptuous indifference?

If I could forget—lose consciousness of those ineptitudes which show me too plainly what I am; forget the helpless ending of all hope, the hollow emptiness that fills the place in me, of friendship, love, and truth.

Hastily I stepped up to the door and pulled the bell. But instead of a vigorous peal the wire creaked far in its bearings. I stood looking at the door—the paint had long lost all trace of its original color; it was covered with marks, chipped here and there in flakes, worn through where it had been rubbed and kicked by entering feet.

A curious gate, but it opened. In the dark passage I saw a woman, a little child was hanging to her gown; another smaller still she carried in her arms; from far beyond there came the sound of crying.

"Is Mr. Smith in?"

I looked with interest at this porteress of the porch. She was fastening her dress at the neck, her hair came straggling over her face, and her gown was shabby; but her form, strong and substantial, had a touch of antique grace. Comely but unanimated features surmounted her deep bosom. Her eyes moved slowly towards me like the placid eyes of a browsing ox, as she answered me with deliberation.

"He's left here."

"Where has he gone?"

"Jenny," she called out, "where's Mr. Smith's trunk gone to?"

"Don't know," returned Jenny's voice.

Here was an end of my inquiry, but I asked—"Had he much to do?"

"I don't know that any one came to see him, though perhaps he let them in himself."

"Did he say anything to you?"

"He wasn't the kind that talks much; he kind of kept to himself."

"Do you remember anything he said?"

"Well, he did say as I was a better one in his line than he was himself, but I guess that was one of his ways."

"What sort of man was he?"

"Oh, nothing in particular; you sort of felt you'd seen him before. He'd a black beard and kind of looked you through."

Jenny's voice came from the upper landing. "His trunk ain't gone yet." In obedience to a howl from below, she moved off to little Harry or Susan or Jane or one of the swarm.

As I walked away I was glad that I had avoided meeting some seedy individual ready to spout his nonsense. But it was not clear to me what he meant by his signboard. And what did he mean by his remark to the woman? Was it a joke at his unsuccess? He said she taught better than he did—taught unlearning better. What did he mean? Unlearning, forgetting! Why, I thought, of course the woman is an infinitely better professor of the art than he could ever be. For what is childhood save a vast forgetting, all the piled-up pack of cards, the theories, aims, strifes, emulations of a generation cast down in one sweep of oblivion, no vestige remaining. Yes, Mr. Smith is capable of a shrewd remark. Is it not as if mankind longed for the same thing that I do, and sought for it from woman? Forgetting, wiping out, the capacity of beginning again. Remorse wiped out, the ignominy of being sunk lower than any conceivable degree of unmanliness—this faded completely away! Is this then the secret of the tendency of man to woman—the longing for forgetfulness of the race? She, the soft Lethe, wherein all errors are forgotten? Love, the desire of forgetting? Are we but falsely what we think we are, this wrapped-up system of membranes, arteries, vessels but a secondary part of life, the real thing that passing on, that birth and re-birth, the proceeding to happy oblivion after oblivion: and man an eddy in the current, a loitering, a delay, a complicated error, a worked-up stage with a horrible power of mismanagement, himself a mistake, his chief passion to be forgotten? Strange that most reasons for wishing to forget come through women, and woman is the means of forgetting—woman her own antidote. But I had rather see what Smith has to say.

In Search of the Unlearner

On my left was the grey sea; before, behind, and on my right stretched an unbroken waste of sand, level and smooth where it approached the

ocean, but beyond the last line of the sweep of the storm rising in little grass-covered hills. All around was monotonous and still. The clouds hung low in a great darkening veil, and the thick, blurred air was laden with fog. It was one of those days on which Nature makes herself all alike, letting all the jarring differences fall away and presenting an image of rest and peace. The only breaks in the great sameness were the furrows left by the ripples of the falling tide, and myself, a black speck, troubling the immensity. The marks on the sand would soon be swept into oblivion by the rising ocean in its advance. But I—well! If it were to happen, no one in the world would care, no wheel of business or friendship would be hampered—so little was there in my life that I had come fifty miles out of my way, had given up a day and accepted the prospect of a night at a miserable boarding-house, in order to meet an individual from whom I could by no possibility get any good, who probably was an impudent pedant. I had sent to his landlady for Mr. Smith's address, on the eve of starting for a journey to the south had received a scarcely legible scrawl, in which she informed me that the professor was at the little out-of-the-way fishing village which I had just left. I had arrived in the morning; singularly enough the little steamer which set me down at this remote spot bore two other passengers for the same place. We had exchanged words. One of them was an artist who had his sketching things with him, a man of a singularly refined and pleasing expression; a brow beautifully moulded; and a mobile countenance, but worn—a bohemian evidently, who mingled the delights of art with the chance pleasures of a careless crowd. The other was a contrast in every way; he was robust and square cut, carefully dressed, speaking in a slow, deliberate manner, with an air of thought about him which was contradicted by the paraphernalia he put into the boat that took us off—the sample cases of a commercial traveller. We all three came to the boardinghouse, which was the only accommodation the village afforded. I made inquiries as to Mr. Smith's presence, and learnt that he had gone out early that morning, intending to walk to a still smaller village farther along the coast and return in the evening. Accordingly I had determined to go out and meet him on his return journey. I had no fear of not recognizing him or not meeting him; there was but one path, and the sandy vista of coast presented no figures but those of fishermen near at hand waiting by their boats.

The path left the level sands and wandered up and down amongst

the sand hills—hills produced by nothing but the ceaseless winds, but made permanent by the coarse and straggling grass. At a turn in the winding path I suddenly came face to face with the man I sought. He was tall, dark, and walked rapidly; his eyes were keen and piercing, but he did not look at me till, standing directly in his path, I said, "Mr. Smith, I believe." With a gesture of his arm he arrested my hand, which I had involuntarily raised; he looked me in the face and said, "Excuse me, we have not met before." This was not the indigent professor of a purposeless art whom I had expected to meet; the glance was full of concentration and energy—the glance of a man accustomed to seize an occasion, not of a waif and dreamer, but of one who directed the destinies of others. Still it must be the man I sought, so I continued, "I called on you in New York, attracted by the singularity of your notice."

"In what do you consider it singular?" he asked.

"Unlearner," I said, "that is not usual."

"Some mischievous boy, no doubt, has put an 'un' before a word of a very usual significance."

"The significance may be common enough, but 'learner' is never seen on a door plate; the more usual designation is 'teacher'—learner in such a position has no significance."

"In that case you do not better it much by putting 'un' before it."

"It is perfectly intelligible," I replied; "in composition the word learner retains its primitive significance of an importer of learning, and unlearner is one who relieves others of the burdens of knowledge."

"Admitting that," he replied, "a man may profess elocution or Greek in New York and yet not be willing to impart instruction wherever he goes."

"Certainly," I answered, "but as I am not likely to see you again, perhaps you will satisfy a natural curiosity on my part as to the nature of the profession of which you are the most eminent representative."

"You do me too much honor," he replied; "I am at the very bottom of the calling."

"To do justice to both our views, in brief, you are the only representative of the profession."

"You are right," he answered, "unless you would include in it those who contribute their services by teaching something in an inefficient manner."

"Omitting those who hardly count, I can understand that the demands on your services are so incessant that you hardly care to interrupt your brief period of rest," I said.

"Indeed, I do not find myself much in request—I would gladly welcome a competitor or two, if there were more demand for the kind of service I render; it is I that have to seek, not I that am sought. But what is it you wish to know?"

There was, if not in his words, at any rate in his expression of countenance, an air of aloofness and superiority which I judged inappropriate coming from a struggling teacher, so I said—"There is a certain absence of display and ostentation about your profession which is commendable. A dancing master, a teacher of the flute, in offering to instruct tacitly lays claim to the possession of skill in dancing or flute playing, whereas you, in offering to aid in unlearning, lay claim to proficiency and ability in nothing."

"I am quite willing to admit," he answered, "that you possess intelligence, if that is the object of your remarks, but at present I am not exercising my profession."

"Perhaps your indisposition to exercise your profession has something to do with my possession of intelligence," I interrupted.

His manner changed—for the first time he looked me full in the face; I felt as if I was called on for a combat. Not that there was anything unfriendly in his look, he might be on my side or against me, but it was a face that took away from me in a moment any inclination for trifling insincerity or pretense—it was like the face of battle.

"Are you sure," he said, "that you want to unlearn? Look at the sea. From here we can see a multitude of small waves; if we were on a high eminence we should see the larger ocean billows on whose surface merely these small disturbances are. From a still greater height we should see the great wave of the tide, whose great sweep might mean life or death to a swimmer, buffetting the tumultuous little waves. Is it not the greater tides that you should strive to learn, forgetting the momentary disturbances?

"No," I said, "that is more learning, not less—and there need be no forgetting in that learning, for from a close and intimate knowledge of the little waves the larger movements could be discovered, depending as they do on the larger movements."

"And yet," he replied, "it is the path to that which you seek, for in unlearning, as well as in everything else, there is a certain heedless-

ness and recklessness which defeats its own end, a desire of grasping the all which lays hold on nothing."

"There is no need for you to tell me that," I answered, as my past thrilled through me.

"Then wherefore forget? What you have been is the food on which your soul lives. Think how closely connected memory and self-consciousness are; snap the last chord of recollection and you would lose the sense of personal identity."

"I do not want these stale moralities," I said, "we are fettered and bound by the past, and oblivion—utter oblivion—is a cheap price to pay for freedom."

"So you know that you are fettered and bound, but you have got to learn that you act in doing nothing; you do not see where freedom lies. But have you ever lived? For life is where man takes up the work of nature and forms a network of close personal knowledge, linking each to each, preparing that body in which the soul of man lives. Three people came to me once, whose destinies were influenced by each other. One of them, a farmer in New England, had committed a murder. Up to forty years of age he had lived among the people of his little village, as one of them, as in these rural communities they carry on their lives. But he fell into difficulties, and, going one day to the house of his largest creditor, he found him inexorable. A quarrel arose; there was no one about. He killed the man, and taking a sum of money, sufficient to relieve him from his difficulties, he went away unperceived. The murder was attributed to roving vagrants. But the farmer found a sense of isolation and aloofness creeping over him. At last, when he could bear it no longer, he went to a neighbor and told him of his crime. 'Least said the soonest mended,' said he, and counselled the man to keep silence. 'But,' said the murderer, 'I want you to help me; you, to whom it does not matter, can be quite open and perfectly known by every one, then I being linked on to you and you to them, I shall be joined again.' But his neighbor hesitated—there were several things quite inconsistent with his position as a deacon which prevented him from acceding to this request. Being moved by the man's eagerness he explained everything fully to him and finally convinced him that what he asked was impossible. The murderer let some time go by till, finding himself sinking still deeper in his isolation, he told a young and lovely girl of what he had done. She was very sorry, and told him that he must live a very good life ever afterwards. 'But I

want you to help me,' he said; 'it will not cost you anything. I want you to be perfectly open, so that every one knows everything about you; then you, knowing everything about me, I shall be linked on.'

"'But,' she exclaimed, 'I could not tell John all about James,' and pitying the man she told him all about herself. He, seeing that he was unreasonable, went away, and after a time gave himself up, and was executed. Then his neighbor felt a new earnestness and cheerfulness come over him, and the girl—she was married then—threw her arms round her husband and in a flood of tears felt a weight and oppression removed—he had done for them what he had wanted them to do for him."

[The artist and the other man, named Clement, join the narrator and the Unlearner; and the two latecomers tell their life stories. The Unlearner starts giving advice.]

His next words were addressed to Clement: "You belong to a modern school which finds reality in the fulness of the relations in which a thing is; you do not conceive it as existing apart from its relations. Now, as the events and circumstances in which you know an individual disappear in time, you cannot believe that he continues to exist. But why should you say that the events and circumstances which are past in time exist no longer, making your consciousness a measure of existence? Thought is a path which it is difficult to retrace. You must go on. You believe in the permanence of matter, the conservation of energy. Take the next step and recognize the conservation of events. Every event which you experience is a permanent thing, altering but always existing. Think of yourself, this is the conception of the soul for you, as always existing in every act and circumstance of your life, so that you—your complete self in your whole life—are continually changing and altering, the present being that part on which you are now engaged. In this way you can come into substantial agreement with the one who is to be your life's companion."

"But," said Clement, "there isn't room for any more than I can see in space."

"Here you introduce a conception which has been made for utility into a discussion in which we have need of certainty. You cannot observe anything which is not analogous to an activity of your own. For

purposes of use you have gained an intuition, by means of which you observe the world in space. If you want to know more of existence, you must not take the conceptions of the arts and manufactures without criticism, but you must form a higher intuition by means of which you can observe more."

"I see what you mean," said Clement; "though I have always before dismissed such notions as chimerical." Then turning to me, with that conviction of his words being valuable which so seldom leaves a man who has gained his living by talking, he said—"It is perfectly possible that what we experience as fleeting, passing events in time should be permanent, altering things. Imagine this piece of wood"—he took up a piece of driftwood a few inches long—"passing through a sheet of paper, but paper of a curious kind, which closed up round all the irregularities of the wood. A creature living on the paper, and having no experience beyond it, would think of the successive sections of the piece of wood as a piece of matter in his limited world, while the face of the piece of wood would appear to him as a series of changes affecting the contour of the section. He only knows the surface as a series of changes. Now if the piece of wood is altering itself, that surface which, perceiving it under his conditions, he calls a fleeting series of events may be changing and altering, and similarly——"

Nothing was to be gained by continuing a discussion of this kind. Mr. Smith had refused to do anything for the artist, and he was now trying to devise a modus vivendi for Clement—a matter which did not interest me at all. So I turned to him and said:—

"I shall be stopping on this coast for a few days; perhaps I shall have the opportunity of meeting you again."

He answered me by an inclination of his head.

"Do you know," I continued, "if I can find accommodation in the village farther on?"

"Yes," he answered; "there is one house where you can make yourself comfortable, if you do not object to simple fare. You will find it a very interesting place."

"I will go on there. May I ask you," I added, turning to Clement, "to have the kindness to tell the people on your return to send my things after me?"

"I'll see to that," said he.

"Before many days are over I will meet you again," said the Unlearner, and we turned in opposite directions.

A Warning

Dark clouds hung around, hastening on the obscurity of night; the little footpath went wandering in and out between the sand hills, and wearied of it, I left it, striking far out to where the line of the sea was visible. Walking on and approaching nearer to this sure guide, I wondered what would happen if, pressing on and on, I entered the looming waters, and, passing beyond the possibility of return, were swallowed up. Were I to be dropped out of the world really and for ever, who would care? Some people, perchance, would faintly wonder what had become of me. But none of the wheels of business or of life would be stopped--the irrevocable end of all would come, closing upon a life of wasted opportunities, of heedlessness, fraught with bitter pain to others, of real disappointment irrevocably suffered, for death has placed its seal on them. How is it and wherefore that I have been landed in such an unreal world, one in which I could welcome any greed, any passion, as a god giving me life, and wiping away the dull emptiness that fills all things?

There must be reality somewhere. But all I have ever sought has been fictitious--sham knowledge was all I learnt and empty aims were all I conceived. And when I began to live in the world I could not distinguish feigned love: I did not understand the signs of deceit and mockery. Rather, was I not a deceit and mockery myself? Forget--if I could but forget! No atonement, no re-doing possible! What I envy most is these sands, whose miserable furrows are washed away by tomorrow's tide. Oblivion, whose image is the great, dark wall of cloud and shifting, barren sand--that is what I long for, the only possible beginning.

A singularly clear and musical cry came to me. I looked, but saw nothing. The immense waste stretched to right and left, on and behind. Far on my left was the white line of the living waters, far, almost undistinguished, on my right were the low sand hills of the shore. From somewhere between me and them came the cry. It was repeated, and at length I discerned a figure far in shore--though whether it were a woman or a boy, I could not tell.

Turning, I began to walk towards the creature who, starting out of this barren immensity, hailed me, and wished to speak to me. But it did not await me; it came towards me, making signs and movements, from which I gathered that I was to retrace my steps. I went back, always tending in shore, and it moved parallel with me. At length it

beckoned to me, and approaching directly I saw that it was a girl. "The sands are very dangerous out there," she said, "this is your best way"; and keeping some distance in front of me, she led me to the path which ran among the sand hills. Almost before I could thank her, she had turned off along another trodden way, and was lost to sight.

[In the village, the narrator takes a room in the house of a fisherman who turns out to be the guardian of the girl, named Natalia, who warned him of the quicksand. He begins to teach Natalia what he knows of the world, and kindles in her a desire to go and see it for herself. As she is a foundling, rather than a native of the village, no one objects when he buys her clothes and a steamer ticket to the city. He has fallen in love with her, although he does not fully realize this.]

An Unexpected Result

No picture of solitude can be greater than that of the long, fading line which lingers still when the vessel that left it marked on the sky has vanished. The steamer was gone, bearing Natalia to her new life, carrying her to her destination. At last every vestige of the long, thin line had disappeared. I arose and turned, uncertain where to go. I wanted to see the place again where she had lived, to talk to some of the little fellows with whom she had talked so often, to hear their unconscious repetition of her words. I turned and walked by the sea. It was growing dusk; the tide was low. In that level, monotonous waste I guided myself as I walked by the dim meeting of land and water; the rising wind blew chill.

I had walked for an hour or so, when I thought I heard a cry; it was like the musical cry with which she had hailed me—it must have been about at the same place. I looked around; there was no one there. Suddenly about my feet came the foam of a wave; the wind whistled, all the sea was a sheet of foam.

Trying to reach the dry land, I came on a treacherous place and sank up to my knees. I extricated myself with difficulty. Then, striving to remember the directions she had given me, I began to retrace my steps. But the waves were upon me—waves such as one could not hope to swim through; a storm was blowing, and all the sea was a sheet of foam. I struggled on for some distance, when all at once the

sand seemed to suck my feet down. I felt that I must make haste if I did not want to get into serious danger. Resting my body on the water, I freed my feet, and found my way to a firmer place. But, proceeding, the waves suddenly seemed to me higher. I had come to a depression in the sand, and at last one wave went over my head.

For some seconds I held my breath, then I felt the air again, but at the moment I was breathing a long roll of foam came. My mouth was full of water. Choosing my time, I tried to get a breath from beyond the foam by leaping up from the bottom, but I only gained momentary inhalations, and before long my breath refused to be controlled. Water—there was nothing but water; it was in my eyes, in my ears, over my head. But I became calmer, not more excited, fighting my way step by step, when I could find the ground in the direction in which I judged the shore lay. "This predicament," I thought, "will be a good one to think about when I am safe by my own fireside in the evenings. It will make me relish safety and comfort." A larger wave, which gathered without these awful foaming tops, lifted me up. I saw that I had been struggling along instead of inshore. Then it was all a period of confused gasps and determined struggles, I becoming all the while calmer and more sure of reaching the shore by steady swimming. I gave up trying to keep my head above. I trusted to chance, and swam steadily through the water, with a slow, methodical stroke. I seemed to make hardly any exertion at all about it. "How strange it is," I thought, "that I have not seen the boat that will rescue me, provided I cannot reach the shore myself."

Then, perfectly plainly and clearly, though I knew I was struggling in the water, I heard my father's voice: "Come, old boy," he said, "and sit on my knee." I saw the fire burning, the tea things were on the table. At that moment I felt the air again, gasped a great breath, then down—down. At my side I felt there was a little girl in black—yes, it was she, my sister; we were following my brother to the grave. "So both of us are lost," I thought; "poor mother, if she had known this was to be the end of all!" Then I became aware of the overpowering pressure of the water; my body seemed to hang limp and flaccid. How do I know which is the direction of the land? What is the use of struggling? All I can do is to keep my mouth shut and trust to the waves throwing me on shore.

My mother's hand was on my shoulder—I had my hat in my hand—there were crowds of people. I had on my new blue knickerbockers; there was some dust on my shoes, as I sat down I

wiped it off. There was a crowd of well-dressed persons—why, I was in church! But there was a dimness about it all; instead of being there I was riding with my father in a car. There was an old horse with a big head toiling and toiling along outside the window, but always going backward. Then, as in a dream, I saw a downcast face—Gretchen's; how I longed to know what the look on her face meant! But the scene changed to a street in New York and a curious notice on a door. Then I was walking with Natalia—the steamer went and then—why, here really I was drowning!—the water deep, deep above me. "At any rate she will be with good friends," I thought; "she will be cared for"; and I was thankful for my forethought. Suddenly it came over me that this was what the unlearner meant, saying he would meet me here. And thinking thus, I, too, pass beyond life, awaiting, like all the others, what meets us there.

* * * * *

All the vagueness as of a dream passed away. I was actually with my father, sitting by his side. Through a window I saw a horse's head jogging up and down. The horse was trotting; I could see the motion of every step—yet it was going backward all the while. I asked my father—we sitting in the street car together—"Why does that horse go backward when he is going forward?" But my father was reading his paper and did not answer me. I went on looking with amazement at the horse going backwards, though he was trotting on. Then my father said, "We'll get out here." The car stopped, and I saw with satisfaction the horse go on, dragging his load quite fast after him.

* * * * *

Then I find myself lying down, trying to go to sleep in the woods. I know Scotchy is near me, though I cannot see him. We call him Scotchy because he comes from Scotland; he is at the technical school with me. There is no doubt about my being really with him, though it seems only a moment from the last scene. I know my father is dead and my mother is very poor; my uncle sends me money every month. Scotchy is lying wrapped up in his thick cloak; I'm in my waterproof blanket. The sounds of the night woods are mysterious around us, full of a nameless influence. Out of them comes distinctly the noise of a larger animal moving. The footstep of the fox is of all sounds that I know the most tense upon the ear. The pit-pat of his footfalls comes so quick and regular, then it stops. I lie perfectly still, but the animal waits, suspicious. Then he comes on again and stops. When he moves the footfalls come regularly, mechanically; then they stop absolutely,

suddenly. I feel his readiness for sudden flight, for sudden spring also. In the dark stillness of the night I do not know what size I am, and there close beside me is fear and ferocity, changing into one another so quickly. It might be a tiger, and I a hind. From nothing so quickly as from a footfall spreads Nature's contagion of fear or fury. At length in the silence I can bear it no longer.

"Scotchy," I say, "are you asleep?"

"No."

"Tell me something to break the silence."

Scotchy says: "I will tell you about Anstruther, who was ruined in the bank failure; all his people were ruined together; there was no hope of recovery. He did not sit down to repine, but went into a foundry as an apprentice. He settled down, hoping to become a regular hand, getting, if he were lucky, in the course of time his two pounds a week. One day, as he was filing at his bench, a party of ladies and gentlemen came through the shop. When one of them was opposite his bench, she looked at him. It was the old look and the treasured smile. She put out her dainty gloved hand for him to shake, but he could not take it in his oily hand. She said, 'I am so glad you are going bravely to work,' and then the party of visitors moved on. Soon after that there came a parcel to his lodgings. Opening it, he found—it was her writing—a little cylinder; a round, oblong piece of iron. It was quite plain and simple, accurately turned; but nothing, not designed for anything, except it might be a paperweight. So he used it for a paperweight.

"But presently the piece of iron began to come in his dreams. It occurred persistently. She came, too, by flashes and gleams, seeming to point with meaning in her gesture to the round piece of iron. In his dream he took the cylinder in his hand, weighing it, and it seemed lighter than it should be if it were solid iron. Waking, he remembered his dream, and looked with interest at the iron keeping a few pieces of paper down. It was a simple object enough, yet night after night it came persistently, she always appearing, also bidding him do something, and pointing to the piece of iron—she with her old smile, and a touch of new, winning, beseeching loveliness. At last, in his dream, he put the thing in a lathe, and turned the end off. Out of it, for it was hollow, began to drop rubies and pearls and precious stones of many kinds; it was full of jewels, inestimable, rare jewels. Always with the jewels falling out his dream ended."

"Did he weigh the real paperweight in his hand Scotchy?" I asked.

"Yes, he did."

"Was it light?"

"Yes, it was."

"Did he turn the end off?"

"No, he knew the workman who made it had cored it out with sand; he saw them every day at the works putting a core of sand inside their coating to save metal."

"Is there any ending to the story?"

"Yes—he did not see her ever again."

* * * * *

I pass suddenly and vividly into another night. I am sitting with my arm round Nellie's waist, her head is leaning against my shoulder; sometimes I pass my other hand through her long hair, sometimes I press her face to mine. In the New England village where my mother and I are living all the people are engaged in making boots. I am talking about my prospects. I was living at home because my uncle had fallen into difficulties, and could send me no more money. "Nellie," I say, "I will get a place in the factory; then we can get a cottage of our own."

"Yes, Teddie," she says, "father can get you a place where he's foreman—"

"Oh! I shan't go there, because I'm going to be foreman myself, and I won't turn him out."

"Brave, kind Teddie, it won't turn father out, will it?" she says.

Then the clock strikes one, and she lets me out. That evening is followed by other evenings, and all alike are filled with the delicious clasp of her soft waist, till one evening I say, "I'm not going to be a shoemaker all my life," and she bids me goodbye at two o'clock, crying and sobbing.

* * * * *

I find myself in a little room. It is a lodging; I know it well. My clothes are all torn, and there is rage in my heart, for a farmer in New Jersey set his dog at me when I only tried to sell him a book. On my table is a letter; I take it up. My mother writes to tell me Nellie is married; she breaks it to me gently, but I know she is glad; she never liked Nellie.

* * * * *

I am sitting in a room upstairs; it is a large, well-lighted room, with bookshelves all around. I know that I am in Cambridge, living with my mother in a house that we have taken, while I go to the

University. A brother of my father's has left us all his money. Much of my time I spend with my mother, the rest over my books. There is a kind of difference and aloofness I feel with the other boys. I am just opening a great parcel, reflecting that it is better to buy books than to try and sell them. I have no friend like Scotchy; I am wishing that I had.

* * * * *

It is a dark, cloudy night, in which but a little space overhead the beams of the street lamps are lost. I am stepping briskly along, carrying a cane. The pavement seems elastic, a breeze sweeps up from Central Park, while in my ears floats the music I have just heard, changing itself into scenes and the rapid passing of figures. Before me are the spacious courts of heaven, wide with the width of a sunny landscape, gleaming with the white illumination of marble. Across the great court comes a heavenly messenger, a boy, bearing on his candid lips a message. Two of the blest souls who were sitting there, speaking on some high theme, look up when he approaches. The one is St. Paul, the other I know is St. Simeon Stylites. For his sake the tide of high argument breaks and repeats itself. Beneath the heavenly peace of St. Paul's grand face you still can see the trace of his spiritual wrestling, the marks of the intellectual athlete, the all-encompassing, dominating mind. The far-reaching vision St. Paul was explaining to him had occupied the whole of St. Simeon's attention, and the puzzled, almost anxious, look on his face as he raised it towards the messenger reminded one, though it was heaven, of what his look on earth must have been.

"There is some one come to be judged," said the boy, "a woman."

"Who is she?" asked St. Paul. "What does St. Peter say?"

"St. Peter," the messenger replied, "does not know anything about her, she did not come that way."

"Then she is not a mortal; Simeon, shall we see her?"

"What is she like?" asked St. Simeon.

"I cannot tell," said the heavenly messenger; "she is covered all over and hides her face. She bears a bundle in her arms."

"I think," said St. Simeon, "we can judge her, Paul; it seems to be some penitent; to veil her face is good."

A figure drew near with a mean robe flung all about her, and in her arms a great bundle. But her walk was as the walk of a humbled queen, and in her voice there was the ripple of the waters, the sighing

of the winds, the song of the birds, as she prayed, "Judge me, I have stolen these."

"'Tis well," said St. Paul, "that thou bringest them back. What are they?"

She stooped, rested the bundle on heaven's floor, and opened it. The mean, worn covering was folded back, and there was nothing and yet everything—everything that men have seen of color in the sunset or in the deep sky. There were the grace of the dappled limbs of the fawn, the lines of strength of the tiger, the wonderful green of the forests, the all-burying forests in their wonderful mazes, the delicate blue of the distance, the depths of the ocean, the semblance and likeness of everything there has been on earth. There, without the substance and body of them, were the grace and beauty of human countenances, the bloom on the cheeks, the vermeil lips, the glance of loving, passionate, ardent, alluring eyes, and the quiet, long, still gaze of dark eyes. There was the glamour and grace and beauty of all that man has ever loved to gaze upon—the tendril-crowned boy Bacchus, in his radiant appeal to the eye, was there, though he was not. There were the flash of white limbs through translucent water, the raised arms of Venus, her head waving like a flower between them. All was there; not the substance of things, but the show of them—all color, all sights; and the wonderful-voiced woman spoke. Her speaking was like a song, like all the music that ever sounded, like all the sounds that ever were, so rich and full and deep it was—calming, soothing, passion-arousing, awakening, mocking, loving, enticing—the cadence of wind-swept forests, the laughing of a girl, all were in it as she said, "All these are not mine, and I have taken them, all the sounds of my voice, and these I have brought here. Henceforth I will be mute and without all these. Oh, judge me."

St. Simeon hid his face in his hands, for he had never thought to see that in heaven which he had avoided as much as in him lay on earth. But St. Paul, the ardent tent-maker, the endurer of toils, familiar with many climes and men, looked at the mingled show of the wonderful bundle, awake, alert, and ready to inquire how the question that arose should be settled in accordance with his system. He did not listen to St. Simeon, who exclaimed, with still averted eyes, "Thrice happy man! Oh, cast all down into the deepest hell, that, there being no more of those on earth, men may save their souls."

But asked wherefore, these being not hers, she still had taken them,

bedecked herself with them, "It was mine," she answered, "at the creation to keep the busy atoms dancing, to turn and twist them on their moving course, playing the shuttle of vibrations in all the system. But men wove robes and garments, inventing light and color, placing light and color and sound before me. They praised me, calling me Nature and wonderful, beautiful. I, because I liked their praise, put on these robes that were none of mine, making pretense to be as they would care to see me; all that you see I put on, feigning to be what men praised—I, who all the while have no part in any of these things, whose it is to move the atoms on their ceaseless wheeling."

St. Paul was silent, not for the moment discerning how the law and sin bore on all this, when the greater light of a Presence made them know that another was there.

For thus it is in heaven, if perplexity or doubt assail any of the blest spirits, such is the order of that happy world, the higher spirits there, that dwell in greater illumination and glory of light, reveal themselves, and at their words all lies plain and clear.

To Nature standing suppliant there, begging for judgment and peace, with all that made her fair in man's eyes surrendered by her, the Judge of all spoke: "Child, know that, because thou caredst for man's praise, there is that in thee wherewith thou canst be to him all he longs for. Leave here these feigned garments and the voice he has lent thee; go thy way, be to him as thy awakening heart tells thee."

Nature turned and went. On earth a new day began. The light of dawn, the sunset of even, no longer were what man put on her, but were of Nature's own. In all the visible world, in all the joys and beauties of the earth, she began to be herself, not clothed in the feigned robes she wore before, in which, because man had woven them, there was of his evil.

In that new day no more was there deception in the joys men clasped, no longer did the glamour hide an emptiness within, no rose fell crumbling and withered the moment man pressed it to his lips. No longer in all his joys did man perpetually grasp his own imaginations and beyond them—emptiness; no longer did he chase the mask of pleasure for its own sake, seeking but himself. For behind all joys, all delights, was she filling his soul more full of her, there herself squandering each hour more pleasures than he had ever dreamed of, each almost unmarked, each lost and overwhelmed in the unfolding glory of her awakening heart.

"Very pretty, white limbs flashing through the water is good," says a voice by my ear.

There by my side is my intimate friend, whom I know better than I do any one else.

"It has nothing to do with you. I was thinking of an escape from you," I say.

"And so you made a pretty picture to please me. Lord, what a time we would have in that renovated world!"

And the creature puts out his tongue, licking like an animal licks round its chops—a perfectly beast-like, unconscious gesture—then he goes a step or two ahead, wheels round and faces me.

"Foul brute, shall I never escape you?"

"You don't want to," he says, with an insinuating grin on his animal countenance.

"Keep your place at any rate. I have thoughts you have nothing to do with. You touch and influence me but by a corner of myself."

"We won't quarrel about that," he says. "We are old friends, you know, and you are always thinking about me, really, however much you may seem to go after other things."

He is still standing opposite me. I look straight in his face—a thing I seldom do—looking at him searchingly, inquiringly.

"Ah! But I remember you different," I say, as I look, "or rather there were two of you, who came to me together; you, yes, but you were different when the other was with you, you were different. He had so quiet and intense a look on his uplifted face, it was as if he saw an angel. I knew you both, when I felt the heavenly wonder that Nellie should speak to me the day we met outside her father's gate."

The low face of the creature before me looks perplexed and troubled, as if some human feeling could penetrate him too. "Yes," he says, "there were two of us, but he hadn't my vitality." Then, with a leer, he adds, "Come with me; I am left you."

"To heel, dog!" I say, and he comes crouching after me. As I walk, I see one and another of his kind following or arm in arm with men like myself, and I know that the city is full of them. There comes by a wretched man in rags, blear-eyed, his intimate beside him, so distorted and deformed, I wonder he can crawl. The two are quarrelling. I hear the man say, "You've brought me to this, have you? But see what a fine thing I've made of you."

I turn to him who is following me, saying, "I, calm-lipped, self-con-

trolled, can let all pass wherein you live. I belong to that band which strive for objects you know nothing of."

"Making yourself endlessly wretched," he returns. "You know you have no joy in life save for me, and all you think or do is to give me pleasure."

"It is not so," I say. But he is close to me; he takes my arm familiarly; I know I shall never be rid of him, and do not want to.

* * * * *

"A singular talent you have for making inappropriate observations," says Paget in his drawling voice. "I know them well. The worthy pair of burghers you have just pointed out as living a simple, patriarchal life have just signed an agreement for their simple Gretchen, whom you appreciate so highly, at Hoffner's Colosseum. Their other girl was a great disappointment to them, she went lame; but they count on this one."

I do not doubt Paget. He employs his leisure, which is ample, in the study of the *personnel* of the Viennese music halls, but I cannot help remarking: "A pure, innocent girl like that! It is impossible that they mean to give her over to that life."

"They know what they are about. They have been very careful of her; it is faces like that which are most charmingly provocative in the right setting; they lose their value if they begin too young. Come over; I'll introduce you."

We join the trio. The man looks like a respectable subofficial in a Government department. But he has no reserve in discussing his daughter's engagement with my friend. Gretchen sits silent, her eyes cast down. It is a pretty, simple face, fresh and charming.

"Madam," I say suddenly, addressing the mother, "cannot you keep your daughter from that place?" She looks at me offended. I am aware of a look of amusement in Paget's eyes. But in Gretchen's, what is it? She glances at me, and in that glance there is something which moves me strangely. What is it, appeal, gratitude, interest? But her eyes are cast down again before I can read anything for certain.

* * * * *

Into my room comes Paget. "Here's the old fox's answer to your proposal; he says he won't accept any assistance from you to help Gretchen to study for a different branch of the profession; and he won't take a sum to enable her to marry. He considers she has good and honorable prospects in the line he has chosen for her, and he

requests you not to see them again, unless you will marry her. He's got an idea—"

"No more of this," I say. "I should be glad to discontinue our acquaintance in future." A feeling of repulsion against these traffickers in maiden innocence and modesty, and against Paget, who had become callous to this kind of thing, comes over me. Gretchen's look is in my eyes—it is appeal, yes, it is a desperate appeal. Paget stands still a moment, then he says, "All right," and goes away. I remain standing—Gretchen my wife! If so, what a horror it would be that she should remain a moment in her present surroundings!

* * * * *

It is my home—this room I find myself in with the fire burning brightly—there Gretchen sitting opposite, looking at it, like me. Her face is an enigma to me still; she does not care to speak in English, and my German does not interest her. All at once, as I am watching her, I hear a voice speaking to her. It is he—that other part of myself that I call my friend. "Gretchen," I say, "it was not I who spoke to you just now."

"There is no one else here."

"Do you think that was I myself?"

"Yes, of course."

"Gretchen," I exclaimed, "it was to save you from such as he that I married you."

"Ich verstehe nicht was du meinest—Liebst du mich nicht?"

"Why, Gretchen, don't you know the kind of love I love you with? I love your happiness, your good, first of all; I would have done anything to make you happy. If it would have made you happy you should have had any money you liked without me."

Her eyes gleam for a moment with a happiness, a gleam as of an imagined happiness, such as I had never seen them shine with before. Then she turns to me and says: "Ach! Scherze doch nicht. Die Männer sind alle gleich."

The sudden illumination of her face brings it home to me; I might have known it any moment before. There was a young actor whom I had sometimes seen at her parents' house—it was he whom she loved. She had married me, as she would have danced before the footlights, because she must get money.

And I know that this friend, this part of me, had fooled me; it was he who had directed every move that led me to my marriage, when I thought my motives were so different. You can blame me, you who

have never been fooled by this Pan creature, who weaves the exhalations of earth into the shapes he wills, who assumes the garb of pity, duty, sacrifice, speaks in the name of utility, common sense, sanctity, and whatever he finds will gain his ends.

In a moment I know I am to Gretchen but the man who bought her, deprived her of that spark of love at which she warmed her little hands, which might have become a flame and irradiated her whole life.

There are moments when you realize how absolutely true all your friends say of you is, how false is the impression of yourself you have been living under. I know that the reasons my acquaintances assigned for my marriage were the true ones. Gretchen was a simple little beauty whose charms would certainly have made her a success in her destined profession. Because she had looked at me as a girl cannot help looking at a man who takes an interest in her—no, because my friend had led me, had taken possession of me, I had married her; had very likely spoiled her life too, for she had conceived the idea that, because she gave up the man she fancied, she would have every luxury money could give her.

* * * * *

"Now he's putting his frills on," says Adela Stephenson.

Opposite her is sitting her husband, my old friend Scotchy, concocting in an impressive manner in the tin brazier a welsh rabbit of his own invention, in which bananas replace the cheese.

"He must be a great assistance to you, Mrs. Stephenson."

"Oh, he is."

"I think," said Stephenson, "we might have sherry"—he is evidently deliberating with what fluid he could replace beer.

"I think I said sherry," he repeated, looking at her.

"It is in the cupboard," she replied, with an ostentatiously unconcerned air.

With the air of a martyr he gets up, finds the sherry, pours in a glassful, and proceeds with his stirring.

"If anything gets very desperate," says Adela, continuing her conversation with me, "I call him in. Louie was desperately naughty yesterday; she's his niece, not mine, so I told him he must punish her. I had punished her enough myself, for she always says something to make me feel mean when I punish her, and I felt very small, for she had been very naughty. So Robert said he would punish her. I told him she would say something to make him feel small, but he was nobly willing. He told her she was to go without her orange for a

week at breakfast—she always has one. She said, 'Many children don't have an orange ever.' I don't think he was a bit more successful in his punishments than I was."

Walking away, I reflect that, because of me, Gretchen will never tell any one he's putting his frills on, or its equivalent in German. It doesn't do me good, seeing the little nook Stephenson lives in. What do I care for all the aims or institutions of men? Those with whom I would join, in a desire for rational good, what do they do but push me down into the void and emptiness of my union with Gretchen? All good objects and good people—much more likely I care for bad objects and people: they at least promise me freedom. What good did all that ever I learned do me? It is all barren words, artificial, all a sham, making me so that I cannot know a sham from a true thing, even when it is myself. I catch sight of a notice with a curious word on it. It is the board with UNLEARNER written on it. I approach.

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. . . I see the look in Natalia's eyes as she listens, see the cottage walls recede as she lives in the palace-home the poets have made for such as she.

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. . . The last thread of smoke has vanished; how long it hung suspended in the air! The very last sign and token of her now gone, and solitude enfolding from the air, the sky, the sea wrapping in folds and folds upon my heart.

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. . . The waves are over me; I lie moving in the rushing waters, a sodden, inert mass. My meager existence is over; nothing achieved, nothing done; empty, worthless. Was it worth while that my father and mother should have tended me, loved me, cherished me, for this? The bright dewdrops of the spring of life shone; how different from this I thought I was, but it is over now.

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Stay, what is this sudden surprise? Just as truly as I am lying here, rising and sinking with the heave of the waters, so I am in each of the scenes and places I have ever been in, living and acting in them. It has been coming over me in scenes which I thought were vivid memories, but now I know they are actual presences. I am a child again with other children. I see my father: each step, each act, each little thing, I go through again, living the very life I lived before. I am a man with other men and women. I am moving, speaking with them, and they with each other—not only I, but all are living as they ever lived. My whole life has become to me as my body was in the life just finished.

In any part of my life I can be with whomsoever I will: I walk with my little sister, I talk with Scotchy. But an oppressive bond is on me; I do exactly the same things I did before; I say the same things. I cannot get out of the chain of events; cannot say, "How strange it is, Scotchy, that I should have died, and yet you and I be talking here!" Yet though the bond—the fetter of unalterability—is on me, all feeling of loss and of the irrevocable passes away, for all are here; once together I know for ever together.

But watching closely with so eager a curiosity, I see that each of us is not doing exactly the same—and see, our lives are altering.

And a new consciousness comes over me. I see that, like everything else in Nature, our lives are altering, developing, our whole lives in every event and circumstance. I see my life suddenly transformed from the pitiful thing it is. I see that it is changing—the whole of it. It is the body of a higher will, changing, moving, altering in a new direction wherein death does not lie. 'Tis life, indeed, for what may not my life become? I feel that sudden touch which Nature lays on all those that die, saying to them, "Know! I am ever changing, altering. With me everything is in a state and stage of development. I allow not anything to be cast in a rigid mould, not even thy past life in thy imagination."

And the will that acts along the whole line, the will whose body is the whole life—that I catch, fragmentarily present here and there in my life—that will, shown, not in great things, but in minute, almost invisible changes, that will is what I prize and treasure, for it is the means whereby my life alters, the means by which it is what it has become.

But I long to pass from this wide consciousness; for, while I have it, I am not in the work of altering my life. To do that I must give up this wide view, and, plunging in it part by part, let all else of my life save the present seem like mere memory or expectation.

To this I pass. Meanwhile I see that once I did not fall into my error, and again I shall not. From one state to another I proceed, and in my life, as it now is, I welcome all its life-long burdens as a step upon the way, the necessary attendants upon a progression. My aversion to all the piled-up fabric of human thought, level with the clouds, reaching the foundations of the earth, I understand it well; was not Natalia hidden from me then? For all thought, all questioning of the unseen, is but a step towards her whose soul moves with ours; all is unable to be understood by those incapable of love. Well did Dante picture wisdom as, for him, the light of the countenance of her who spelt her name really and actually with a B and E. The cold, keen blade of the in-

tellect is beaten from the sorrows of lives on lives. It is only as life after life—each differing from the last in virtue of the will, that has the whole of life for its body—that we in life can picture our true destiny.

And in this brief vision between life and life, in which the soul sees how from life to life events mould and shape themselves, I see that my life has not ever been as it was in this one course. All is slowly altering from life to life, and in my higher consciousness I see wherefrom, whereunto it moulds itself. I see how each little thing is different, and how in just this, that now I have lived, I grasp the realized results of ages of the higher transverse growth. Once I did not turn back, when the steamer took Natalia on its evanescent water-path, but went with her, leading her to a life which her intolerant spirit could not brook. I know that once I held her hand in death, and she, unmoved by life's failure, turned to me and smiled a look of hope. That lives within me, and I feel that horror of betraying her noble innocence, which is all that remains in life's consciousness of the ghastly ruins of an error in the past. Now I know that I have attained, attained so much as makes me nearer, many a step nearer, to my life with Natalia; for it is only in the world-regard in the care for all life that souls can walk together perfectly, and only so now will Natalia and I walk together. And we shall walk together. Were it not so, I would not tell you this. For we ourselves are larger than the limited life we think is all, and that most holy union of soul and soul, not in one of our fleeting lives, not in many, is it attained. The great personal ends of the world pass over fleeting lives and lives, each life giving us that task which, unless done faithfully, is an inseparable barrier, which, accomplished, is a step. And maybe for a period, as I pass again and again in life through the changes that we in life's concentration think are all, I may not see Natalia. But I know she awaits me. How long it will be I know not, but each moment of silent earnestness, each trace of that great will which alters all, in all my life, I prize and worship, for it brings me nearer her.

And Gretchen, too, I see her growing—feel that the refusal to bend her instinct of love to time and circumstance is slowly awakening, will gather strength, and sometime she will not be the creature of her parents' wish.

Behind the visions of my unfinished, ended life, I see the figure of the Unlearner, not standing as he did that day upon the sands, but receding, becoming larger, more and more remote, till he is like that space which lies beyond aught we can ever think of, and he seems to say: "Thou shalt attain at last, but so much must first be done."

A NEW ERA OF THOUGHT

[This selection contains excerpts of *A New Era of Thought* (1888), including material from the Introductory Note and Chapters 7, 9, 10 and 11.]

Introductory Note

At the completion of a work, or at the completion of the first part of a work, the feelings are necessarily very different from those with which the work was begun; and the meaning and value of the work itself bear a very different appearance. It will therefore be the simplest and shortest plan, if I tell the reader briefly what the work is to which these pages are a guide, and what I consider to be its value when done.

The task was to obtain a sense of the properties of higher space, or space of four dimensions, in the same way as that by which we reach a sense of our ordinary three-dimensional space. I now prefer to call the task that of obtaining a familiarity with higher matter, which shall be as intuitive to the mind as that of ordinary matter has become. The expression "higher matter" is preferable to "higher space," because it is a somewhat hasty proceeding to split this concrete matter, which we touch and feel, into the abstractions of extension and impenetrability. It seems to me that I cannot think of space without matter, and therefore, as no necessity compels me to such a course, I do not split up the concrete object into subtleties, but I simply ask: "What is that which is to a cube or block or shape of any kind as the cube is to a square?"

In entering upon this inquiry we find the task is twofold. Firstly, there is the theoretical part, which is easy, viz. to set clearly before us the relative conditions which would obtain if there were a matter physically higher than this matter of ours, and to choose the best means of liberating our minds from the limitations imposed on it by the particular conditions under which we are placed. The second part

of the task is somewhat laborious, and consists of a constant presentation to the senses of those appearances which portions of higher matter would present, and of a continual dwelling on them, until the higher matter becomes familiar.

The reader must undertake this task, if he accepts it at all, as an experiment. Those of us who have done it, are satisfied that there is that in the results of the experiment which make it well worthy of a trial.

And in a few words I may state the general bearings of this work, for every branch of work has its general bearings. It is an attempt, in the most elementary and simple domain, to pass from the lower to the higher. In pursuing it the mind passes from one kind of intuition to a higher one, and with that transition the horizon of thought is altered. It becomes clear that there is a physical existence transcending the ordinary physical existence; and one becomes inclined to think that the right direction to look is, not away from matter to spiritual existences, but towards the discovery of conceptions of higher matter, and thereby of those material existences whose definite relations to us are apprehended as spiritual intuitions. Thus, "material" would simply mean "grasped by the intellect, become known and familiar." Our apprehension of anything which is not expressed in terms of matter, is vague and indefinite. To realize and live with that which we vaguely discern, we need to apply the intuition of higher matter to the world around us. And this seems to me the great inducement to this study. Let us form our intuition of higher space, and then look out upon the world.

Secondly, in this progress from ordinary to higher matter, as a general type of progress from lower to higher, we make the following observations. Firstly, we become aware that there are certain limitations affecting our regard. Secondly, we discover by our reason what those limitations are, and then force ourselves to go through the experience which would be ours if the limitations did not affect us. Thirdly, we become aware of a capacity within us for transcending those limitations, and for living in the higher mode as we had lived in the previous one.

We may remark that this progress from the ordinary to the higher kind of matter demands an absolute attention to details. It is only in the retention of details that such progress becomes possible. And as, in this question of matter, an absolute and unconventional examination gives us the indication of a higher, so, doubtless, in other questions, if we but come to facts without presupposition, we begin to know that

there is a higher and to discover indications of the way whereby we can approach. That way lies in the fullness of detail rather than in the generalization.

Biology has shown us that there is a universal order of forms or organisms, passing from lower to higher. Therein we find an indication that we ourselves take part in this progress. And in using the little cubes we can go through the process ourselves, and learn what it is in a little instance.

But of all the ways in which the confidence gained from this lesson can be applied, the nearest to us lies in the suggestion it gives—and more than the suggestion, if inclination to think be counted for anything—in the suggestion of that which is higher than ourselves. We, as individuals, are not the limit and end-all, but there is a higher being than ours. What our relation to it is, we cannot tell, for that is unlike our relation to anything we know. But, perhaps all that happens to us is, could we but grasp it, our relation to it.

At any rate, the discovery of it is the great object beside which all else is as secondary as the routine of mere existence is to companionship. And the method of discovery is full knowledge of each other. Thereby is the higher being to be known. In as much as the least of us knows and is known by another, in so much does he know the higher. Thus, scientific prayer is when two or three meet together, and, in the belief of one higher than themselves, mutually comprehend that vision of the higher, which each one is, and, by absolute fullness of knowledge of the facts of each other's personality, strive to attain a knowledge of that which is to each of their personalities as a higher figure is to its solid sides.

Self Elements in Our Consciousness

It is often taken for granted that our consciousness of ourselves and of our own feelings has a sort of direct and absolute value.

It is supposed to afford a testimony which does not require to be sifted like our consciousness of external events. But in reality it needs far more criticism to be applied to it than any other mode of apprehension.

To a certain degree we can sift our experience of the external world, and divide it into two portions. We can determine the self elements and the realities. But with regard to our own nature and emotions, the discovery which makes a science possible has yet to be made.

There are certain indications, however, springing from our observation of our own bodies, which have a certain degree of interest.

It is found that the processes of thought and feeling are connected with the brain. If the brain is disturbed, thoughts, sights, and sounds come into the consciousness which have no objective cause in the external world. Hence we may conclusively say that the human being, whatever he is, is in contact with the brain, and through the brain with the body, and through the body with the external world.

It is the structures and movements in the brain which the human being perceives. It is by a structure in the brain that he apprehends nature, not immediately. The most beautiful sights and sounds have no effect on a human being unless there is the faculty in the brain of taking them in and handing them on to the consciousness.

Hence, clearly, it is the movements and structure of the minute portions of matter forming the brain which the consciousness perceives. And it is only by models and representations made in the stuff of the brain that the mind knows external changes.

Now, our brains are well furnished with models and representations of the facts and events of the external world.

But a most important fact still requires its due weight to be laid upon it.

These models and representations are made on a very minute scale—the particles of brain matter which form images and representations are beyond the power of the microscope in their minuteness. Hence the consciousness primarily apprehends the movements of matter of a degree of smallness which is beyond the power of observation in any other way.

Hence we have a means of observing the movements of the minute portions of matter. Let us call those portions of the brain matter which are directly instrumental in making representations of the external world—let us call them brain molecules.

Now, these brain molecules are very minute portions of matter indeed; generally they are made to go through movements and form structures in such a way as to represent the movements and structures of the external world of masses around us.

But it does not follow that the structures and movements which they perform of their own nature are identical with the movements of the portions of matter which we see around us in the world of matter.

It may be that these brain molecules have the power of four-dimensional movement, and that they can go through four-dimensional movements and form four-dimensional structures.

If so, there is a practical way of learning the movements of the very small particles of matter—by observing, not what we can see, but what we can think.

For, suppose these small molecules of the brain were to build up structures and go through movements not in accordance with the rule of representing what goes on in the external world, but in accordance with their own activity, then they might go through four-dimensional movements and form four-dimensional structures.

And these movements and structures would be apprehended by the consciousness along with the other movements and structures, and would seem as real as the others—but would have no correspondence in the external world.

They would be thoughts and imaginations, not observations of external facts.

Now, this field of investigation is one which requires to be worked at.

At present it is only those structures and movements of the brain molecules which correspond to the realities of our three-dimensional space which are in general worked at consistently. But in the practical part of this book it will be found that by proper stimulus the brain molecules will arrange themselves in structures representing a four-dimensional existence. It only requires a certain amount of care to build up mental models of higher space existences. In fact, it is probably part of the difficulty of forming three-dimensional brain models, that the brain molecules have to be limited in their own freedom of motion to the requirements of the limited space in which our practical daily life is carried on.

Note: For my own part I should say that all those confusions in remembering which come from an image taking the place of the original mental model—as, for instance, the difficulty in remembering which way to turn a screw, and the numerous cases of images in thought transference—may be due to a toppling over in the brain, four-dimensionalwise, of the structures formed—which structures would be absolutely safe from being turned into image structures if the brain molecules moved only three-dimensionalwise.

It is remarkable how in science "explaining" means the reference of the movements and tendencies to movement of the masses about us to the movements and tendencies to movement of the minute portions of matter.

Thus, the behavior of gaseous bodies—the pressure which they

exert, the laws of their cooling and intermixture are explained by tracing the movements of the very minute particles of which they are composed.

Another View of the Aether

We have supposed in the case of a plane world that the surface on which the movements take place is inactive, except by its vibrations. It is simply a smooth support.

For the sake of simplicity let us call this smooth surface "the aether" in the case of a plane world.

The aether then we have imagined to be simply a smooth, thin sheet, not possessed of any definite structure, but excited by real disturbances of the matter on it into vibrations, which carry the effect of these disturbances as light and heat to other portions of matter. Now, it is possible to take an entirely different view of the aether in the case of a plane world.

Let us imagine that, instead of the aether being a smooth sheet serving simply as a support, it is definitely marked and grooved.

Let us imagine these grooves and channels to be very minute, but to be definite and permanent.

Then, let us suppose that, instead of the matter which slides in the aether having attractions and repulsions of its own, that it is quite inert, and has only the properties of inertia.

That is to say, taking a disk or a plane world as a specimen, the whole disk is sliding on the aether in virtue of a certain momentum which it has, and certain portions of its matter fit into the grooves in the aether, and move along those grooves.

The size of the portions is determined by the size of the grooves. And let us call those portions of matter which occupy the breadth of a groove, atoms. Then it is evident that the disk sliding along over the aether, its atoms will move according to the arrangement of the grooves over which the disk slides. If the grooves at any one particular place come close together, there will be a condensation of matter at that place when the disk passes over it; and if the grooves separate, there will be a rarefaction of matter.

If we imagine five particles, each slipping along in its own groove, if the particles are arranged in the form of a regular pentagon, and the grooves are parallel, then these five particles, moving evenly on, will

maintain their positions with regard to one another, and a body would exist like a pentagon, lasting as long as the grooves remained parallel.

But if, after some distance had been traversed by the disk, and these five particles were brought into a region where one of the grooves tended away from the others, the shape of the pentagon would be destroyed, it would become some irregular figure. And it is easy to see that if the grooves separated, and other grooves came in amongst them, along which other portions of matter were sliding, that the pentagon would disappear as an isolated body, that its constituent matter would be separated, and that its particles would enter into other shapes as constituents of them, and not of the original pentagon.

Thus, in cases of greater complication, an elaborate structure may be supposed to be formed, to alter, and to pass away; its origin, growth, and decay being due, not to any independent motion of the particles constituting it, but to the movement of the disk whereby its portions of matter were brought to regions where there was a particular disposition of the grooves.

Then the nature of the shape would really be determined by the grooves, not by the portions of matter which passed over them—they would become manifest as giving rise to a material form when a disk passed over them, but they would subsist independently of the disk; and if another disk were to pass over the same grooves, exactly the same material structures would spring up as came into being before.

If we make a similar supposition about our aether along which our earth slides, we may conceive the movements of the particles of matter to be determined, not by attractions or repulsions exerted on one another, but to be set in existence by the alterations in the directions of the grooves of the aether along which they are proceeding.

If the grooves were all parallel, the earth would proceed without any other motion than that of its path in the heavens.

But with an alteration in the direction of the grooves, the particles, instead of proceeding uniformly with the mass of the earth, would begin to move amongst each other. And by a sufficiently complicated arrangement of grooves it may be supposed that all the movements of the forms we see around us are due to interweaving and variously disposed grooves.

Thus the movements, which any body goes through, would depend on the arrangement of the aethereal grooves along which it was passing. As long as the grooves remain grouped together in approximately

the same way, it would maintain its existence as the same body; but when the grooves separated, and became involved with the grooves of other objects, this body would cease to exist separately.

Thus the separate existences of the earth might conceivably be due to the disposition of those parts of the aether over which the earth passed. And thus any object would have to be separated into two parts, one the aethereal form, or modification which lasted, the other the material particles which, coming on with blind momentum, were directed into such movements as to produce the actual objects around us.

In this way there would be two parts in any organism, the material part and the aethereal part. There would be the material body, which soon passes and becomes indistinguishable from any other material body, and the aethereal body which remains.

Now, if we direct our attention to the material body, we see the phenomena of growth, decay, and death, the coming and the passing away of a living being, isolated during his existence, absolutely merged at his death into the common storehouse of matter.

But if we regard the aethereal body, we find something different. We find an organism which is not so absolutely separated from the surrounding organisms—an organism which is part of the aether, and which is linked to other aethereal organisms by its very substance—an organism between which and others there exists a unity incapable of being broken, and a common life which is rather marked than revealed by the matter which passes over it. The aethereal body moreover remains permanently when the material body has passed away.

The correspondences between the aethereal body and the life of an organism such as we know, is rather to be found in the emotional region than in the one of outward observation. To the aethereal form, all parts of it are equally one; but part of this form corresponds to the future of the material being, part of it to his past. Thus, care for the future and regard for the past would be the way in which the material being would exhibit the unity of the aethereal body, which is both his past, his present, and his future. That is to say, suppose the aethereal body capable of receiving an injury, an injury in one part of it would correspond to an injury in a man's past; an injury in another part—that which the material body was traversing—would correspond to an injury to the man at the present moment; injury to the aethereal body at another part, would correspond to injury coming to the man at

some future time. And the self-preservation of the aethereal body, supposing it to have such a motive, would in the last case be the motive of regarding his own future to the man. And inasmuch as the man felt the real unity of his aethereal body, and did not confine his attention to his material body, which is absolutely disunited at every moment from its future and its past—inasmuch as he apprehended his aethereal unity, insomuch would be care for his future welfare, and consider it as equal in importance to his present comfort. The correspondence between emotion and physical fact would be that the emotion of regard corresponded to an undiscerned aethereal unity. And then also, just as the two tips of two fingers put down on a plane would seem to a plane being to be two completely different bodies, not connected together, so one and the same aethereal body might appear as two distinct material bodies, and any regard between the two would correspond to an apprehension of their aethereal unity. In the supposition of an aethereal body, it is not necessary to keep to the idea of the rigidity and permanence of the grooves defining the motion of the matter which, passing along, exhibits the material body. The aethereal body may have a life of its own, relations with other aethereal bodies, and a life as full of vicissitudes as that of the material body, which in its total orbit expresses in the movements of matter one phase in the life of the aethereal body.

But there are certain obvious considerations which prevent any serious dwelling on these speculations—they are only introduced here in order to show how the conception of higher space lends itself to the representation of certain indefinite apprehensions—such as that of the essential unity of the race—and affords a possible clue to correspondences between the emotional and the physical life.

The whole question of our relation to the aether has to be settled. That which we call the aether is far more probably the surface of a liquid, and the phenomena we observe due to surface tensions. Indeed, the physical questions concern us here nothing at all. It is easy enough to make some supposition which gives us a standing ground to discipline our higher-space perception; and when that is trained, we shall turn round and look at the facts.

The conception which we shall form of the universe will undoubtedly be as different from our present one, as the Copernican view differs from the more pleasant view of a wide immovable earth beneath a vast vault. Indeed, any conception of our place in the

universe will be more agreeable than the thought of being on a spinning ball, kicked into space without any means of communication with any other inhabitants of the universe.

Higher Space and Higher Being

In the instinctive and sense perception of man and nature there is all hidden, which reflection afterwards brings into consciousness.

We are conscious of somewhat higher than each individual man when we look at men. In some, this consciousness reaches an extreme pitch, and becomes a religious apprehension. But in none is it otherwise than instinctive. The apprehension is sufficiently definite to be certain. But it is not expressible to us in terms of the reason.

Now, I have shown that by using the conception of higher space it is easy enough to make a supposition which shall show all mankind as physical parts of one whole. Our apparent isolation as bodies from each other is by no means so necessary to assume as it would appear. But, of course, a supposition of that kind is of no value, except as showing a possibility. If we came to examine into the matter closely, we should find a natural relationship which accounted for our consciousness being limited as at present it is.

The first thing to be done, is to organize our higher-space perception, and then look. We cannot tell what external objects will blend together into the unity of a higher being. But just as the riddle of the two hands becomes clear to us from our first inspection of higher space, so will there grow before our eyes greater unities and greater surprises.

We have been subject to a limitation of the most absurd character. Let us open our eyes and see the facts.

Now, it requires some training to open the eyes. For many years I worked at the subject without the slightest success. All was mere formalism. But by adopting the simplest means, and by a more thorough knowledge of space, the whole flashed clear.

Space shapes can only be symbolical of four-dimensional shapes; and if we do not deal with space shapes directly, but only treat them by symbols on the plane—as in analytical geometry—we are trying to get a perception of higher space through symbols of symbols, and the task is hopeless. But a direct study of space leads us to the knowledge

of higher space. And with the knowledge of higher space there come into our ken boundless possibilities. All those things may be real, whereof saints and philosophers have dreamed.

Looking on the fact of life, it has become clear to the human mind, that justice, truth, purity, are to be sought—that they are principles which it is well to serve. And men have invented an abstract devotion to these, and all comes together in the grand but vague conception of Duty.

But all these thoughts are to those which spring up before us as the shadow on a bank of clouds of a great mountain is to the mountain itself. On the piled-up clouds falls the shadow—vast, imposing, but dark, colorless. If the beholder but turns, he beholds the mountain itself, towering grandly with verdant pines, the snowline, and the awful peaks.

So all these conceptions are the way in which now, with vision confined, we apprehend the great existences of the universe. Instead of an abstraction, what we have to serve is a reality, to which even our real things are but shadows. We are parts of a great being, in whose service, and with whose love, the utmost demands of duty are satisfied.

How can it not be a struggle, when the claims of righteousness mean diminished life—even death—to the individual who strives? And yet to a clear and more rational view it will be seen that in his extinction and loss, that which he loves—that real being which is to him shadowed forth in the present existence of wife and child—that being lives more truly, and in its life those he loves are his for ever.

But, of course, there are mistakes in what we consider to be our duty, as in everything else; and this is an additional reason for pursuing the quest of this reality. For by the rational observance of other material bodies than our own, we come to the conclusion that there are other beings around us like ourselves, whom we apprehend in virtue of two processes—the one simply a sense one of observation and reflection—the other a process of direct apprehension.

Now, if we did not go through the sense process of observation, we might, it is true, know that there were other human beings around us in some subtle way—in some mesmeric feeling; but we should not have that organized human life which, dealing with the things of the world, grows into such complicated forms. We should for ever be good-humored babies—a sensuous, affectionate kind of jelly-fish.

And just so now with reference to the high intelligences by whom we are surrounded. We feel them, but we do not realize them.

To realize them, it will be necessary to develop our power of perception.

The power of seeing with our bodily eye is limited to the three-dimensional section.

But I have shown that the inner eye is not thus limited; that we can organize our power of seeing in higher space, and that we can form conceptions of realities in this higher space, just as we can in our ordinary space.

And this affords the groundwork for the perception and study of these other beings than man. Just as some mechanical means are necessary for the apprehension of our fellows in space, so a certain amount of mechanical education is necessary for the perception of higher beings in higher space.

Let us turn the current of our thought right round; instead of seeking after abstractions, and connecting our observations by ideas, let us train our sense of higher space and build up conceptions of greater realities, more absolute existences.

It is really a waste of time to write or read more generalities. Here is the grammar of the knowledge of higher being—let us learn it, not spend time in speculating as to whither it will lead us.

Yet one thing more. We are, with reference to the higher things of life, like blind and puzzled children. We know that we are members of one body, limbs of one vine; but we cannot discern, except by instinct and feeling, what that body is, what the vine is. If to know it would take away our feeling, then it were well never to know it. But fuller knowledge of other human beings does not take away our love for them; what reason is there then to suppose that a knowledge of the higher existences would deaden our feelings?

And then, again, we each of us have a feeling that we ourselves have a right to exist. We demand our own perpetuation. No man, I believe, is capable of sacrificing his life to any abstract idea; in all cases it is the consciousness of contact with some being that enables him to make the last human sacrifice. And what we can do by this study of higher space, is to make this consciousness, which has been reserved for a few, the property of all. Do we not all feel that there is a limit to our devotion to abstractions, none to beings whom we love. And to love them, we must know them.

Then, just as our own individual life is empty and meaningless without those we love, so the life of the human race is empty and meaningless without a knowledge of those that surround it. And al-

though to some an inner knowledge of the oneness of all men is vouchsafed, it remains to be demonstrated to the many.

The perpetual struggle between individual interests and the common good can only be solved by merging both impulses in a love towards one being whose life lies in the fulfilment of each.

The Scientific Basis of Altruism and Religion

The reader will doubtless ask for some definite result corresponding to these words—something not of the nature of an hypothesis or a might-be. And in that I can only satisfy him after my own powers. My only strength is in detail and patience; and if he will go through the practical part of the book, it will assuredly dawn upon him that here is the beginning of an answer to his request. I only study the blocks and stones of the higher life. But here they are definite enough. And the more eager he is for personal and spiritual truth, the more eagerly do I urge him to take up the practical work, for the true good comes to us through those who, aspiring greatly, still submit their aspirations to fact, and who, desiring to apprehend spirit, still are willing to manipulate matter.

The particular problem, at which I have worked for more than ten years, has been completely solved. It is possible for the mind to acquire a conception of higher space as adequate as that of our three-dimensional space, and to use it in the same manner.

There are two distinct ways of studying space—our familiar space at present in use. One is that of the analyst, who treats space relations by his algebra, and discovers marvellous relations. The other is that of the observer or mechanician, who studies the shapes of things in space directly.

A practical designer of machines would not find the knowledge of geometrical analysis of immediate help to him; and an artist or draughtsman still less so.

Now, my inquiry was whether it was possible to get the same power of conception of four-dimensional space as the designer and draughtsman have of three-dimensional space. It is possible.

And with this power it is possible for us to design machines in higher space, and to conceive objects in this space, just as a draughtsman or artist does.

Analytical skill is not of much use in designing a statue or inventing a machine, or in appreciating the detail of either a work of art or a mechanical contrivance.

And hitherto the study of four-dimensional space has been conducted by analysis. Here, for the first time, the fact of the power of conception of four-dimensional space is demonstrated, and the means of educating it are given.

And I propose a complete system of work, of which the volume on four space is the first installment.

I shall bring forward a complete system of four-dimensional thought—mechanics, science, and art. The necessary condition is, that the mind acquire the power of using four-dimensional space as it now does three-dimensional.

THE FOURTH DIMENSION

[This selection contains excerpts of *The Fourth Dimension* (1904), including material from Chapters 1, 4 and 5.]

Four-Dimensional Space

There is nothing more indefinite, and at the same time more real, than that which we indicate when we speak of the "higher." In our social life we see it evidenced in a greater complexity of relations. But this complexity is not all. There is, at the same time, a contact with, an apprehension of, something more fundamental, more real.

With the greater development of man there comes a consciousness of something more than all the forms in which it shows itself. There is a readiness to give up all the visible and tangible for the sake of those principles and values of which the visible and tangible are the representation. The physical life of civilized man and of a mere savage are practically the same, but the civilized man has discovered a depth in his existence, which makes him feel that that which appears all to the savage is a mere externality and appurtenage to his true being.

Now, this higher—how shall we apprehend it? It is generally embraced by our religious faculties, by our idealizing tendency. But the higher existence has two sides. It has a being as well as qualities. And in trying to realize it through our emotions we are always taking the subjective view. Our attention is always fixed on what we feel, what we think. Is there any way of apprehending the higher after the purely objective method of a natural science? I think that there is.

Plato, in a wonderful allegory, speaks of some men living in such a condition that they were practically reduced to be the denizens of a shadow world. They were chained, and perceived but the shadows of themselves and all real objects projected on a wall, towards which their faces were turned. All movements to them were but movements on the surface, all shapes but the shapes of outlines with no substantiality.

Plato uses this illustration to portray the relation between true being and the illusions of the sense world. He says that just as a man

liberated from his chains could learn and discover that the world was solid and real, and could go back and tell his bound companions of this greater higher reality, so the philosopher who has been liberated, who has gone into the thought of the ideal world, into the world of ideas greater and more real than the things of sense, can come and tell his fellow men of that which is more true than the visible sun—more noble than Athens, the visible state.

Now, I take Plato's suggestion; but literally, not metaphorically. He imagines a world which is lower than this world, in that shadow figures and shadow motions are its constituents; and to it he contrasts the real world. As the real world is to this shadow world, so is the higher world to our world. I accept his analogy. As our world in three dimensions is to a shadow or plane world, so is the higher world to our three-dimensional world. That is, the higher world is four-dimensional; the higher being is, so far as its existence is concerned apart from its qualities, to be sought through the conception of an actual existence spatially higher than that which we realize with our senses.

Here you will observe I necessarily leave out all that gives its charm and interest to Plato's writings. All those conceptions of the beautiful and good which live immortally in his pages.

All that I keep from his great storehouse of wealth is this one thing simply—a world spatially higher than this world, a world which can only be approached through the stocks and stones of it, a world which must be apprehended laboriously, patiently, through the material things of it, the shapes, the movements, the figures of it.

We must learn to realize the shapes of objects in this world of the higher man; we must become familiar with the movements that objects make in his world, so that we can learn something about his daily experience, his thoughts of material objects, his machinery.

The means for the prosecution of this enquiry are given in the conception of space itself.

It often happens that that which we consider to be unique and unrelated gives us, within itself, those relations by means of which we are able to see it as related to others, determining and determined by them.

Thus, on the earth is given that phenomenon of weight by means of which Newton brought the earth into its true relation to the sun and other planets. Our terrestrial globe was determined in regard to other bodies of the solar system by means of a relation which subsisted on the earth itself.

And so space itself bears within it relations of which we can determine it as related to other space. For within space are given the conceptions of point and line, line and plane, which really involve the relation of space to a higher space.

Where one segment of a straight line leaves off and another begins is a point, and the straight line itself can be generated by the motion of the point.

One portion of a plane is bounded from another by a straight line, and the plane itself can be generated by the straight line moving in a direction not contained in itself.

Again, two portions of solid space are limited with regard to each other by a plane; and the plane, moving in a direction not contained in itself, can generate solid space.

Thus, going on, we may say that space is that which limits two portions of higher space from each other, and that our space will generate the higher space by moving in a direction not contained in itself.

Another indication of the nature of four-dimensional space can be gained by considering the problem of the arrangement of objects.

If I have a number of swords of varying degrees of brightness, I can represent them in respect of this quality by points arranged along a straight line.

If I place a sword at *A*, figure 22, and regard it as having a certain brightness, then the other swords can be arranged in a series along the line, as at *A, B, C,* etc., according to their degrees of brightness.

If now I take account of another quality, say length, they can be arranged in a plane. Starting from *A, B, C,* I can find points to represent different degrees of length along such lines as *AF, BD, CE,* drawn from *A* and *B* and *C* (see fig. 23). Points on these lines represent different degrees of length with the same degree of brightness. Thus the whole plane is occupied by points representing all conceivable varieties of brightness and length.

Bringing in a third quality, say sharpness, I can draw, as in figure 24, any number of upright lines. Let distances along these upright lines represent degrees of sharpness, thus the points *F* and *G* will

FIGURE 22

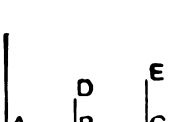


FIGURE 23

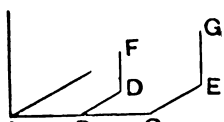


FIGURE 24

represent swords of certain definite degrees of the three qualities mentioned, and the whole of space will serve to represent all conceivable degrees of these three qualities.

If now I bring in a fourth quality, such as weight, and try to find a means of representing it as I did the other three qualities, I find a difficulty. Every point in space is taken up by some conceivable combination of the three qualities already taken.

To represent four qualities in the same way as that in which I have represented three, I should need another dimension of space.

Thus we may indicate the nature of four-dimensional space by saying that it is a kind of space which would give positions representative of four qualities, as three-dimensional space gives positions representative of three qualities.

A Chapter in the History of Four Space

Parmenides, and the Asiatic thinkers with whom he is in close affinity, propound a theory of existence which is in close accord with a conception of a possible relation between a higher and a lower dimensional space. This theory, prior and in marked contrast to the main stream of thought, which we shall afterwards describe, forms a closed circle by itself. It is one which in all ages has had a strong attraction for pure intellect, and is the natural mode of thought for those who refrain from projecting their own volition into nature under the guise of causality.

According to Parmenides of the school of Elea, the all is one, unmoving and unchanging. The permanent amid the transient—that foothold for thought, that solid ground for feeling, on the discovery of which depends all our life—is no phantom; it is the image amidst deception of true being, the eternal, the unmoved, the one. Thus says Parmenides.

But how explain the shifting scene, these mutations of things!

"Illusion," answers Parmenides. Distinguishing between truth and error, he tells of the true doctrine of the one—the false opinion of a changing world. He is no less memorable for the manner of his advocacy than for the cause he advocates. It is as if from his firm foothold of being he could play with the thoughts under the burden of which others labored, for from him springs that fluency of supposition and hypothesis which forms the texture of Plato's dialectic.

Can the mind conceive a more delightful intellectual picture than that of Parmenides, pointing to the one, the true, the unchanging, and yet on the other hand ready to discuss all manner of false opinion, forming a cosmogony too, false "but mine own" after the fashion of the time?

In support of the true opinion he proceeded by the negative way of showing the self-contradictions in the ideas of change and motion. It is doubtful if his criticism, save in minor points, has ever been successfully refuted. To express his doctrine in the ponderous modern way we must make the statement that motion is phenomenal not real.

Let us represent his doctrine.[14]

Imagine a sheet of still water into which a slanting stick is being lowered with a motion vertically downwards. Let 1, 2, 3 (fig. 25), be three consecutive positions of the stick. *A*, *B*, *C*, will be three consecutive positions of the meeting of the stick with the surface of the water. As the stick passes down, the meeting will move from *A* on to *B* and *C*.

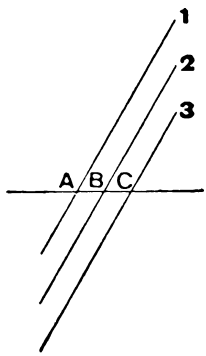


FIGURE 25

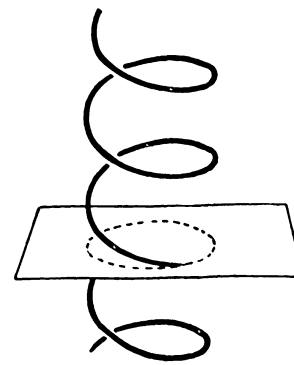


FIGURE 26

Suppose now all the water to be removed except a film. At the meeting of the film and the stick there will be an interruption of the film. If we suppose the film to have a property, like that of a soap bubble, of closing up round any penetrating object, then as the stick goes vertically downwards the interruption in the film will move on.

If we pass a spiral through the film, the intersection will give a point

[14] The representation of Parmenedian idealism in terms of 4-D spacetime is also treated in Kurt Gödel's essay in Schilpp's collection, *Albert Einstein: Philosopher Scientist*. [R.v.B.R.]

moving in a circle shown by the dotted lines (fig. 26). Suppose now the spiral to be still and the film to move vertically upwards, the whole spiral will be represented in the film of the consecutive positions of the point of intersection. In the film the permanent existence of the spiral is experienced as a time series—the record of traversing the spiral is a point moving in a circle. If now we suppose a consciousness connected with the film in such a way that the intersection of the spiral with the film gives rise to a conscious experience, we see that we shall have in the film a point moving in a circle, conscious of its motion, knowing nothing of that real spiral the record of the successive intersections of which by the film is the motion of the point.

It is easy to imagine complicated structures of the nature of the spiral, structures consisting of filaments, and to suppose also that these structures are distinguishable from each other at every section. If we consider the intersections of these filaments with the film as it passes to be the atoms constituting a filmar universe, we shall have in the film a world of apparent motion; we shall have bodies corresponding to the filamentary structure, and the positions of these structures with regard to one another will give rise to bodies in the film moving amongst one another. This mutual motion is apparent merely. The reality is of permanent structures stationary, and all the relative motions accounted for by one steady movement of the film as a whole.

Thus we can imagine a plane world, in which all the variety of motion is the phenomenon of structures consisting of filamentary atoms traversed by a plane of consciousness. Passing to four dimensions and our space, we can conceive that all things and movements in our world are the reading off of a permanent reality by a space of consciousness. Each atom at every moment is not what it was, but a new part of that endless line which is itself. And all this system successively revealed in the time which is but the succession of consciousness, separate as it is in parts, in its entirety is one vast unity. Representing Parmenides' doctrine thus, we gain a firmer hold on it than if we merely let his words rest, grand and massive, in our minds. And we have gained the means also of representing phases of that Eastern thought to which Parmenides was no stranger. Modifying his uncompromising doctrine, let us suppose, to go back to the plane of consciousness and the structure of filamentary atoms, that these structures are themselves moving—are acting, living. Then, in the transverse motion of the film, there would be two phenomena of motion, one due to the reading off in the film of the permanent existences as they are in themselves, and

another phenomenon of motion due to the modification of the record of the things themselves, by their proper motion during the process of traversing them.

Thus a conscious being in the plane would have, as it were, a twofold experience. In the complete traversing of the structure, the intersection of which with the film gives his conscious all, the main and principal movements and actions which he went through would be the record of his higher self as it existed unmoved and unacting. Slight modifications and deviations from these movements and actions would represent the activity and self-determination of the complete being, of his higher self.

It is admissible to suppose that the consciousness in the plane has a share in that volition by which the complete existence determines itself. Thus the motive and will, the initiative and life, of the higher being, would be represented in the case of the being in the film by an initiative and a will capable, not of determining any great things or important movements in his existence, but only of small and relatively insignificant activities. In all the main features of his life his experience would be representative of one state of the higher being whose existence determines his as the film passes on. But in his minute and apparently unimportant actions he would share in that will and determination by which the whole of the being he really is acts and lives.

An alteration of the higher being would correspond to a different life history for him. Let us now make the supposition that film after film traverses these higher structures, that the life of the real being is read off again and again in successive waves of consciousness. There would be a succession of lives in the different advancing planes of consciousness, each differing from the preceding, and differing in virture of that will and activity which in the preceding had not been devoted to the greater and apparently most significant things in life, but the minute and apparently unimportant. In all great things the being of the film shares in the existence of his higher self as it is at any one time. In the small things he shares in that volition by which the higher being alters and changes, acts and lives.

Thus we gain the conception of a life changing and developing as a whole, a life in which our separation and cessation and fugitiveness are merely apparent, but which in its events and course alters, changes, develops; and the power of altering and changing this whole

lies in the will and power the limited being has of directing, guiding, altering himself in the minute things of his existence.

Transferring our conceptions to those of an existence in a higher dimensionality traversed by a space of consciousness, we have an illustration of a thought which has found frequent and varied expression. When, however, we ask ourselves what degree of truth there lies in it, we must admit that, as far as we can see, it is merely symbolical. The true path in the investigation of a higher dimensionality lies in another direction.

The significance of the Parmenidean doctrine lies in this: that here, as again and again, we find that those conceptions which man introduces of himself, which he does not derive from the mere record of his outward experience, have a striking and significant correspondence to the conception of a physical existence in a world of a higher space. How close we come to Parmenides' thought by this manner of representation it is impossible to say. What I want to point out is the adequateness of the illustration, not only to give a static model of his doctrine, but one capable as it were, of a plastic modification into a correspondence into kindred forms of thought. Either one of two things must be true—that four-dimensional conceptions give a wonderful power of representing the thought of the East, or that the thinkers of the East must have been looking at and regarding four-dimensional existence.

And from the numerical idealism of Pythagoras there is but a step to the more rich and full idealism of Plato. That which is apprehended by the sense of touch we put as primary and real, and the other senses we say are merely concerned with appearances. But Plato took them all as valid, as giving qualities of existence. That the qualities were not permanent in the world as given to the senses forced him to attribute to them a different kind of permanence. He formed the conception of a world of ideas, in which all that really is, all that affects us and gives the rich and wonderful wealth of our experience, is not fleeting and transitory, but eternal. And of this real and eternal we see in the things about us the fleeting and transient images.

And this world of ideas was no exclusive one, wherein was no place for the innermost convictions of the soul and its most authoritative assertions. Therein existed justice, beauty—the one, the good, all that the soul demanded to be. The world of ideas, Plato's wonderful creation preserved for man, for his deliberate investigation and their sure

development, all that the rude incomprehensible changes of a harsh experience scatters and destroys.

Plato believed in the reality of ideas. He meets us fairly and squarely. Divide a line into two parts, he says (fig. 27); one to represent the real objects in the world, the other to represent the transitory appearances, such as the image in still water, the glitter of the sun on a bright surface, the shadows on the clouds.

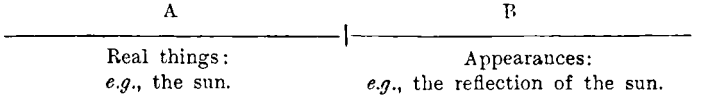


FIGURE 27

Take another line and divide it into two parts (fig. 28), one representing our ideas, the ordinary occupants of our minds, such as whiteness, equality, and the other representing our true knowledge, which is of eternal principles, such as beauty, goodness.

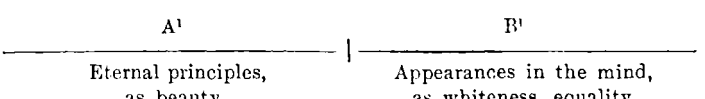


FIGURE 28

Then as A is to B, so is A' to B'.

That is, the soul can proceed, going away from real things to a region of perfect certainty, where it beholds what is, not the scattered reflections; beholds the sun, not the glitter on the sands; true being, not chance opinion.

Now, this is to us, as it was to Aristotle, absolutely inconceivable from a scientific point of view. We can understand that a being is known in the fullness of his relations; it is in his relations to his circumstances that a man's character is known; it is in his acts under his conditions that his character exists. We cannot grasp or conceive any principle of individuation apart from the fullness of the relations to the surroundings.

But suppose now that Plato is talking about the higher man—the four-dimensional being that is limited in our external experience to a three-dimensional world. Do not his words begin to have a meaning? Such a being would have a consciousness of motion which is not as the

motion he can see with the eyes of the body. He, in his own being, knows a reality to which the outward matter of this too solid earth is flimsy superficiality. He too knows a mode of being, the fullness of relations, in which can only be represented in the limited world of sense, as the painter unsubstantially portrays the depths of woodland, plains, and air. Thinking of such a being in man, was not Plato's line well divided?

It is noteworthy that, if Plato omitted his doctrine of the independent origin of ideas, he would present exactly the four-dimensional argument; a real thing as we think it is an idea. A plane being's idea of a square object is the idea of an abstraction, namely, a geometrical square. Similarly our idea of a solid thing is an abstraction, for in our idea there is not the four-dimensional thickness which is necessary, however slight, to give reality. The argument would then run, as a shadow is to a solid object, so is the solid object to the reality. Thus A and B' would be identified.

In the allegory which I have already alluded to, Plato in almost as many words shows forth the relation between existence in a superficies and in solid space. And he uses this relation to point to the conditions of a higher being.

He imagines a number of men prisoners, chained so that they look at the wall of a cavern in which they are confined, with their backs to the road and the light. Over the road pass men and women, figures and processions, but of all this pageant all that the prisoners behold is the shadow of it on the wall whereon they gaze. Their own shadows and the shadows of the things in the world are all that they see, and identifying themselves with their shadows related as shadows to a world of shadows, they live in a kind of dream.

Plato imagines one of their number to pass out from amongst them into the real space world, and then returning to tell them of their condition.

Here he presents most plainly the relation between existence in a plane world and existence in a three-dimensional world. And he uses this illustration as a type of the manner in which we are to proceed to a higher state from the three-dimensional life we know.

It must have hung upon the weight of a shadow which path he took! Whether the one we shall follow toward the higher solid and the four-dimensional existence, or the one which makes ideas the higher realities, and the direct perception of them the contact with the truer world.

The theories which are generally connected with the names of Lobatchewsky and Bolyai bear a singular and curious relation to the subject of higher space.

In order to show what this relation is, I must ask the reader to be at the pains to count carefully the sets of points by which I shall estimate the volumes of certain figures.

No mathematical processes beyond this simple one of counting will be necessary.

Let us suppose we have before us in figure 29 a plane covered with points at regular intervals, so placed that every four determine a square.

Now it is evident that as four points determine a square, so four squares meet in a point.

Thus, considering a point inside a square as belonging to it, we may say that a point on the corner of a square belongs to it and to four others equally: belongs a quarter of it to each square.

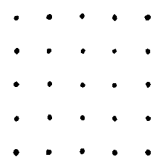


FIGURE 29

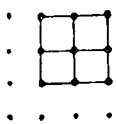


FIGURE 30

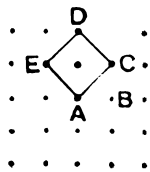


FIGURE 31

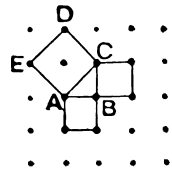


FIGURE 32

Thus the square *ACDE* (fig. 31) contains one point, and has four points at the four corners. Since one-fourth of each of these four belongs to the square, the four together count as one point, and the point value of the square is two points—the one inside and the four at the corner make two points belonging to it exclusively.

Now the area of this square is two unit squares, as can be seen by drawing two diagonals in figure 32.

We also notice that the square in question is equal to the sum of the squares on the sides *AB*, *BC*, of the right-angled triangle *ABC*. Thus we recognize the proposition that the square on the hypotenuse is equal to the sum of the squares on the two sides of a right-angled triangle.

Now suppose we set ourselves the question of determining whereabouts, in the ordered system of points, the end of a line would

come when it turned about a point keeping one extremity fixed at the point.

We can solve this problem in a particular case. If we can find a square lying slantwise amongst the dots which is equal to one which goes regularly, we shall know that the two sides are equal, and that the slanting side is equal to the straight-way side. Thus the volume and shape of a figure remaining unchanged will be the test of its having rotated about the point, so that we can say that its side in its first position would turn into its side in the second position.

Now, such a square can be found in the one whose side is five units in length.

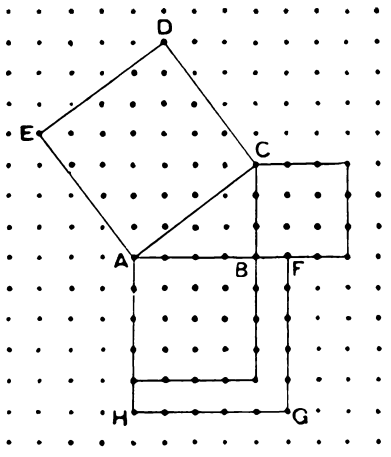


FIGURE 33

In figure 33, in the square on AB, there are—

9 points interior	9
4 at the corners	1
4 sides with 3 on each side, considered as $1\frac{1}{2}$ on each side, because belonging equally to two squares	6

The total is 16. There are 9 points in the square on BC.
In the square on AC there are—

24 points inside	24
4 at the corners ...	1

or 25 altogether.

Hence we see again that the square on the hypotenuse is equal to the squares on the sides.

Now take the square $AFHG$, which is larger than the square on AB. It contains 25 points.

16 inside	16
16 on the sides, counting as	8
4 on the corners	1

making 25 altogether.

If two squares are equal we conclude the sides are equal. Hence, the line AF turning round A would move so that it would after a certain turning coincide with AC.

This is preliminary, but it involves all the mathematical difficulties that will present themselves.

There are two alterations of a body by which its volume is not changed.

One is the one we have just considered, rotation, the other is what is called shear.

Consider a book, or heap of loose pages. They can be slid so that each one slips over the preceding one, and the whole assumes the shape b in figure 34.

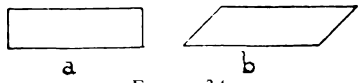


FIGURE 34

The deformation is not shear alone, but shear accompanied by rotation.

Shear can be considered as produced in another way.

Take the square $ABCD$ (fig. 35), and suppose that it is pulled out from along one of its diagonals both ways, and proportionately compressed along the other diagonal. It will assume the shape in figure 36.

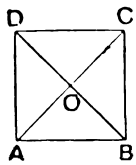


FIGURE 35

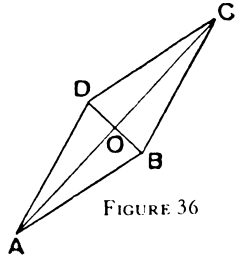


FIGURE 36

This compression and expansion along two lines at right angles is what is called shear; it is equivalent to the sliding illustrated above, combined with a turning round.

In pure shear a body is compressed and extended in two directions at right angles to each other, so that its volume remains unchanged.

Now we know that our material bodies resist shear—shear does violence to the internal arrangement of their particles, but they turn as wholes without such internal resistance.

But there is an exception. In a liquid shear and rotation take place equally easily, there is no more resistance against a shear than there is against a rotation.

Now, suppose all bodies were to be reduced to the liquid state, in which they yield to shear and to rotation equally easily, and then were to be reconstructed as solids, but in such a way that shear and rotation had interchanged places.

That is to say, let us suppose that when they had become solids again they would shear without offering any internal resistance, but a rotation would do violence to their internal arrangement.

That is, we should have a world in which shear would have taken the place of rotation.

A shear does not alter the volume of a body: thus an inhabitant living in such a world would look on a body sheared as we look on a body rotated. He would say that it was of the same shape, but had turned a bit round.

Let us imagine a Pythagoras in this world going to work to investigate, as is his wont.

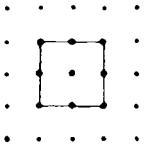


FIGURE 37

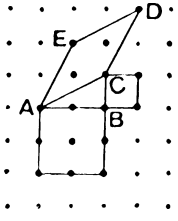


FIGURE 38

Figure 37 represents a square unsheared. Figure 38 represents a square sheared. It is not the figure into which the square in figure 37 would turn, but the result of shear on some square not drawn. It is a simple slanting placed figure, taken now as we took a simple slanting placed square before. Now, since bodies in this world of shear offer no internal resistance to shearing, and keep their volume when sheared,

an inhabitant accustomed to them would not consider that they altered their shape under shear. He would call $ACDE$ as much a square as the square in figure 37. We will call such figures shear squares. Counting the dots in $ACDE$, we find—

2 inside 2
4 at corners 1

or a total of 3.

Now, the square on the side AB has 4 points, that on BC has 1 point. Here the shear square on the hypotenuse has not 5 points but 3; it is not the sum of the squares on the sides, but the difference.

This relation always holds. Look at figure 39.

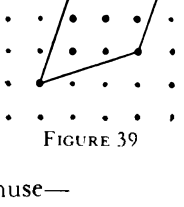


FIGURE 39

Shear square on hypotenuse—

7 internal	7
4 at corners	1
	8

Square on one side—which the reader can draw for himself—

4 internal	4
8 on sides	4
4 at corners	1
	9

The square on the other side is 1. Hence in this case again the difference is equal to the shear square on the hypotenuse, $9 - 1 = 8$.

Thus in a world of shear the square on the hypotenuse would be equal to the difference of the squares on the sides of a right-angled triangle.[15]

[15] It is an interesting exercise in plane geometry to find the general proof of this statement. The trick is to copy Hinton's figure 40, draw in the diagonals of the squares and the parallelogram, and then find two different algebraic expressions for the area of the rectangle formed by connecting the centers of the three quadrilaterals and point A. [R.v.B.R.]

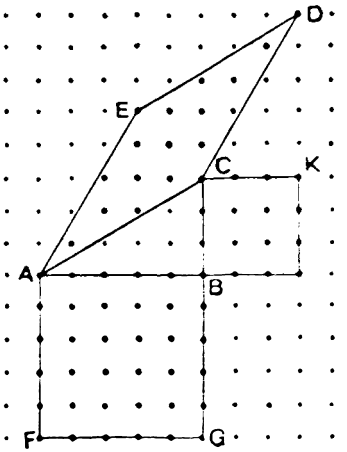


FIGURE 40

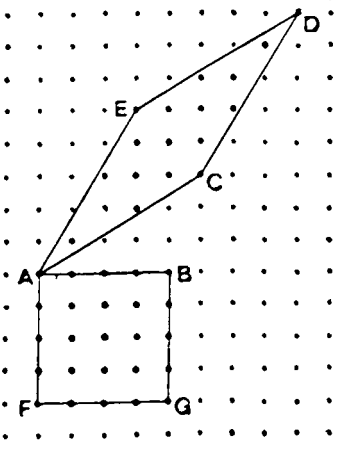


FIGURE 41

In figure 40 another shear square is drawn on which the above relation can be tested.

What now would be the position a line on turning by shear would take up?

We must settle this in the same way as previously with our turning.

Since a body sheared remains the same, we must find two equal bodies, one in the straight way, one in the slanting way, which have the same volume. Then the side of one will by turning become the side of the other, for the two figures are each what the other becomes by a shear turning.

We can solve the problem in a particular case—

In the figure *ACDE* (fig. 41) there are—

15 inside	15
4 at corners	1

a total of 16.

Now in the square *ABGF*, there are 16—

9 inside	9
12 on sides	6
4 at corners	1
	16

Hence the square on *AB* would, by the shear turning, become the shear square *ACDE*.

And hence the inhabitant of this world would say that the line AB turned into the line AC. These two lines would be to him two lines of equal length, one turned a little way round from the other.

That is, putting shear in place of rotation, we get a different kind of figure, as the result of the shear rotation, from what we got with our ordinary rotation. And as a consequence we get a position for the end of a line of invariable length when it turns by the shear rotation, different from the position which it would assume on turning by our rotation.

A real material rod in the shear world would, on turning about A, pass from the position AB to the position AC. We say that its length alters when it becomes AC, but this transformation of AB would seem to an inhabitant of the shear world like a turning of AB without altering in length.[16]

If now we suppose a communication of ideas that takes place between one of ourselves and an inhabitant of the shear world, there would evidently be a difference between his views of distance and ours.

We should say that his line AB increased in length in turning to AC. He would say that our line AF (fig. 33) decreased in length in turning to AC. He would think that what we called an equal line was in reality a shorter one.

We should say that a rod turning round would have its extremities in the positions we call at equal distances. So would he—but the positions would be different. He could, like us, appeal to the properties of matter. His rod to him alters as little as ours does to us.

Now, is there any standard to which we could appeal, to say which of the two is right in this argument? There is no standard.

We should say that, with a change of position, the configuration and shape of his objects altered. He would say that the configuration and shape of our objects altered in what we called merely a change of position. Hence distance independent of position is inconceivable, or practically, distance is solely a property of matter.

There is no principle to which either party in this controversy could appeal. There is nothing to connect the definition of distance with our ideas rather than with his, except the behavior of an actual piece of matter.

For the study of the processes which go on in our world the defini-

[16] In spacetime, the invariant is the square root of the difference of the squares of time and space separation. Hinton here shows us a technique to make this definition natural. [R.v.B.R.]

tion of distance given by taking the sum of the squares is of paramount importance to us. But as a question of pure space without making any unnecessary assumptions, the shear world is just as possible and just as interesting as our world.

It was the geometry of such conceivable worlds that Lobatchewsky and Bolyai studied.

This kind of geometry has evidently nothing to do directly with four-dimensional space.

But a connection arises in this way. It is evident that, instead of taking a simple shear as I have done, and defining it as that change of the arrangement of the particles of a solid which they will undergo without offering any resistance due to their mutual action, I might take a complex motion, composed of a shear and a rotation together, or some other kind of deformation.

Let us suppose such an alteration picked out and defined as the one which means simple rotation; then the type, according to which all bodies will alter by this rotation, is fixed.

Looking at the movements of this kind, we should say that the objects were altering their shape as well as rotating. But to the inhabitants of that world they would seem to be unaltered, and our figures in their motions would seem to them to alter.

In such a world the features of geometry are different. We have seen one such difference in the case of our illustration of the world of shear, where the square on the hypotenuse was equal to the difference, not the sum, of the squares on the sides.

In our illustration we have the same laws of parallel lines as in our ordinary rotation world, but in general the laws of parallel lines are different.

In one of these worlds of a different constitution of matter, through one point there can be two parallels to a given line, in another of them there can be none; that is, although a line be drawn parallel to another it will meet it after a time.

Now it was precisely in this respect of parallels that Lobatchewsky and Bolyai discovered these different worlds. They did not think of them as worlds of matter, but they discovered that space did not necessarily mean that our law of parallels is true. They made the distinction between laws of space and laws of matter, although that is not the form in which they stated their results.

The way in which they were led to these results was the following. Euclid had stated the existence of parallel lines as a postulate—putting

frankly this unproved proposition—that one line and only one parallel to a given straight line can be drawn, as a demand, as something that must be assumed. The words of his ninth postulate are these: "If a straight line meeting two other straight lines makes the interior angles on the same side of it equal to two right angles, the two straight lines will never meet."

The mathematicians of later ages did not like this bald assumption, and not being able to prove the proposition they called it an axiom—the eleventh axiom.

Many attempts were made to prove the axiom; no one doubted of its truth, but no means could be found to demonstrate it. At last an Italian, Sacchieri, unable to find a proof, said: "Let us suppose it not true." He deduced the results of there being possibly two parallels to one given line through a given point, but feeling the waters too deep for the human reason, he devoted the latter half of his book to disproving what he had assumed in the first part.

Then Bolyai and Lobatchewsky with firm step entered on the forbidden path. There can be no greater evidence of the indomitable nature of the human spirit, or of its manifest destiny to conquer all those limitations which bind it down within the sphere of sense than this grand assertion of Bolyai and Lobatchewsky.

Take a line AB and a point C. We say and see and know that through C can only be drawn one line parallel to AB.

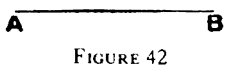


FIGURE 42

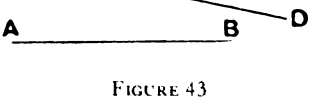


FIGURE 43

But Bolyai said: "I will draw two." Let CD be parallel to AB, that is, not meet AB however far produced, and let lines beyond CD also not meet AB; let there be a certain region between CD and CE, in which no line drawn meets AB. CE and CD produced backwards through C will give a similar region on the other side of C.

Nothing so triumphantly, one may almost say so insolently, ignoring of sense had ever been written before. Men had struggled against the limitations of the body, fought them, despised them, conquered them. But no one had ever thought simply as if the body, the bodily eyes, the organs of vision, all this vast experience of space, had never existed. The age-long contest of the soul with the body, the struggle

for mastery, had come to a culmination. Bolyai and Lobatchewsky simply thought as if the body was not. The struggle for dominion, the strife and combat of the soul were over; they had mastered, and the Hungarian drew his line.

Can we point out any connection, as in the case of Parmenides, between these speculations and higher space? Can we suppose it was any inner perception by the soul of a motion not known to the senses, which resulted in this theory so free from the bonds of sense? No such supposition appears to be possible.

Practically, however, metageometry had a great influence in bringing the higher space to the front as a working hypothesis. This can be traced to the tendency the mind has to move in the direction of least resistance. The results of the new geometry could not be neglected, the problem of parallels had occupied a place too prominent in the development of mathematical thought for its final solution to be neglected. But this utter independence of all mechanical considerations, this perfect cutting loose from the familiar intuitions, was so difficult that almost any other hypothesis was more easy of acceptance, and when Beltrami showed that the geometry of Lobatchewsky and Bolyai was the geometry of shortest lines drawn on certain curved surfaces, the ordinary definitions of measurement being retained, attention was drawn to the theory of a higher space. An illustration of Beltrami's theory is furnished by the simple consideration of hypothetical beings living on a spherical surface (fig. 44).

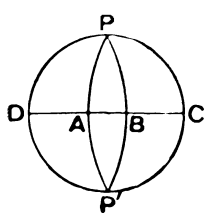


FIGURE 44

Let *ABCD* be the equator of a globe, and *AP*, *BP*, meridian lines drawn to the pole, *P*. The lines *AB*, *AP*, *BP* would seem to be perfectly straight to a person moving on the surface of the sphere, and unconscious of its curvature. Now *AP* and *BP* both make right angles with *AB*. Hence they satisfy the definition of parallels. Yet they meet in *P*. Hence a being living on a spherical surface, and unconscious of its curvature, would find that parallel lines would meet. He would

also find that the angles in a triangle were greater than two right angles. In the triangle *PAB,* for instance, the angles at *A* and *B* are right angles, so the three angles of the triangle *PAB* are greater than two right angles.

Now in one of the systems of metageometry (for after Lobatchewsky had shown the way it was found that other systems were possible besides his), the angles of a triangle are greater than two right angles.

Thus a being on a sphere would form conclusions about his space which are the same as he would form if he lived on a plane, the matter in which had such properties as are presupposed by one of these systems of geometry. Beltrami also discovered a certain surface on which there could be drawn more than one "straight" line through a point which would not meet another given line. I use the word straight as equivalent to the line having the property of giving the shortest path between any two points on it. Hence, without giving up the ordinary methods of measurement, it was possible to find conditions in which a plane being would necessarily have an experience corresponding to Lobatchewsky's geometry. And by the consideration of a higher space, and a solid curved in such a higher space, it was possible to account for a similar experience in a space of three dimensions.

Now, it is far more easy to conceive of a higher dimensionality to space than to imagine that a rod in rotating does not move so that its end describes a circle. Hence, a logical conception having been found harder than that of a four-dimensional space, thought turned to the latter as a simple explanation of the possibilities to which Lobatchewsky had awakened it. Thinkers became accustomed to deal with the geometry of higher space—it was Kant, says Veronese, who first used the expression of "different spaces"—and with familiarity the inevitableness of the conception made itself felt.

From this point it is but a small step to adapt the ordinary mechanical conceptions to a higher spatial existence, and then the recognition of its objective existence could be delayed no longer. Here, too, as in so many cases, it turns out that the order and connection of our ideas is the order and connection of things.

What is the significance of Lobatchewsky's and Bolyai's work?

It must be recognized as something totally different from the conception of a higher space; it is applicable to spaces of any number of dimensions. By immersing the conception of distance in matter to which it properly belongs, it promises to be of the greatest aid in analysis; for the effective distance of any two particles is the product of

complex material conditions and cannot be measured by hard and fast rules. Its ultimate significance is altogether unknown. It is a cutting loose from the bonds of sense, not coincident with the recognition of a higher dimensionality, but indirectly contributory thereto.

Thus, finally, we have come to accept what Plato held in the hollow of his hand; what Aristotle's doctrine of the relativity of substance implies. The vast universe, too, has its higher, and in recognizing it we find that the directing being within us no longer stands inevitably outside our systematic knowledge.

THE RECOGNITION OF THE FOURTH DIMENSION

[This selection is reprinted in its entirety from *Bulletin of the Philosophical Society of Washington*, Vol. 14 (1902).]

There are two directions of inquiry in which the research for the physical reality of a fourth dimension can be prosecuted. One is the investigation of the infinitely great, the other is the investigation of the infinitely small.[1]

By the measurement of the angles of vast triangles whose sides are the distances between the stars, astronomers have sought to determine if there is any deviation from the values given by geometrical deduction. If the angles of a celestial triangle do not together equal two right angles, there would be an evidence for the physical reality of a fourth dimension.

This conclusion deserves a word of explanation. If space is really four dimensional, certain conclusions follow which must be brought clearly into evidence if we are to frame the questions definitely which we put to Nature. If space is four dimensional, there must be a solid material sheet against which we move. This sheet must stretch alongside every object in every direction in which it visibly moves. Every material body must slip or slide along this sheet, not deviating from contact with it in any motion which we can observe.

The necessity for this assumption is clearly apparent if we consider the analogous case of a suppositionary plane world. If there were any creatures whose experience were confined to a plane, we must account for their limitation. If they were free to move in every space direction, they would have a three-dimensional motion; hence they must be physically limited, and the only way in which we can conceive such a limitation to exist is by means of a material surface against which they

[1] This phrasing is probably intended to echo that of the last section of Bernhard Riemann's classic essay, "The Hypotheses Which Lie at the Foundations of Geometry." [R.v.B.R.]

slide. The existence of this surface could only be known to them indirectly. It does not lie in any direction from them in which the kinds of motion they know of leads them. If it were perfectly smooth and always in contact with every material object, there would be no difference in their relations to it which would direct their attention to it.

But if this surface were curved—if it were, say, in the form of a vast sphere—the triangles they drew would really be triangles of a sphere, and when these triangles are large enough the angles diverge from the magnitudes they would have for the same lengths of sides if the surface were plane. Hence by the measurement of triangles of very great magnitude, a plane being might detect a difference from the laws of a plane world in his physical world, and so be led to the conclusion that there was in reality another dimension to space—a third dimension—as well as the two which his ordinary experience made him familiar with.

Now, astronomers have thought it worthwhile to examine the measurements of vast triangles drawn from one celestial body to another with a view to determine if there is anything like a curvature in our space—that is to say, they have tried astronomical measurements to find out if the vast solid sheet against which, on the supposition of a fourth dimension, everything slides is curved or not. These results have been negative. The solid sheet, if it exists, is not curved or, being curved, has not a sufficient curvature to cause any observable deviation from the theoretical value of the angles calculated.

Hence the examination of the infinitely great leads to no decisive criterion. It neither proves nor disproves the existence of a fourth dimension.

Coming now to the prosecution of the inquiry in the direction of the infinitely small, we have to state the question thus: Our laws of movement are derived from the examination of bodies which move in three-dimensional space. All our conceptions are founded on the supposition of a space which is represented analytically by three independent axes and variations along them—that is, it is a space in which there are three independent movements. Any motion possible in it can be compounded out of these three movements, which we may call: up, right, away.

To examine the actions of the very small portions of matter with the view of ascertaining if there is any evidence in the phenomena for the supposition of a fourth dimension of space, we must commence by clearly defining what the laws of mechanics would be on the supposi-

tion of a fourth dimension. It is no use asking if the phenomena of the smallest particles of matter are like—we do not know what. We must have a definite conception of what the laws of motion would be on the supposition of the fourth dimension, and then inquire if the phenomena of the activity of the smaller particles of matter resemble the conceptions which we have elaborated.

Now, the task of forming these conceptions is by no means one to be lightly dismissed. Movement in space has many features which differ entirely from movement on a plane; and when we set about to form the conception of motion in four dimensions, we find that there is at least as great a step as from the plane to three-dimensional space.

I do not say that the step is difficult, but I want to point out that it must be taken. When we have formed the conception of four-dimensional motion, we can ask a rational question of Nature. Before we have elaborated our conceptions we are asking if an unknown is like an unknown—a futile inquiry.

As a matter of fact, four-dimensional movements are in every way simple and more easy to calculate than three-dimensional movements, for four-dimensional movements are simply two sets of plane movements put together. It appears to me one of the most marvelous characteristics of the power of the intellect to find how, without any of the familiarity derived from experience, it is possible to grasp the facts of four-dimensional movement and apprehend the consequences of these conceptions.

Without the formation of an experience of four-dimensional bodies, their shapes and motions, the subject can be but formal—logically conclusive, not intuitively evident. It is to this logical apprehension that I must appeal.

It is perfectly simple to form an experiential familiarity with the facts of four-dimensional movement. The method is analogous to that which a plane being would have to adopt to form an experiential familiarity with three-dimensional movements, and may be briefly summed up as the formation of a compound sense by means of which duration is regarded as equivalent to extension.

Consider a being confined to a plane. A square enclosed by four lines will be to him a solid, the interior of which can only be examined by breaking through the lines. If such a square were to pass transverse to his plane, it would immediately disappear. It would vanish, going in no direction to which he could point.

If, now, a cube be placed in contact with his plane, its surface of

contact would appear like the square which we have just mentioned. But if it were to pass transverse to his plane, breaking through it, it would appear as a lasting square. The three-dimensional matter will give a lasting appearance in circumstances under which two-dimensional matter will at once disappear.

Similarly, a four-dimensional cube, or, as we may call it, a tesseract, which is generated from a cube by a movement of every part of the cube in a fourth direction at right angles to each of the three visible directions in the cube, if it moved transverse to our space, would appear as a lasting cube.

A cube of three-dimensional matter, since it extends to no distance at all in the fourth dimension, could instantly disappear if subjected to a motion transverse to our space. It would disappear and be gone without it being possible to point to any direction in which it had moved.

All attempts to visualize a fourth dimension are futile. If must be connected with a time experience in three space.[18]

The most difficult notion for a plane being to acquire would be that of rotation about a line. Consider a plane being facing a square: If he were told that rotation about a line were possible, he would move his square this way and that. A square in a plane can rotate about a point, but to rotate about a line would seem to the plane being perfectly impossible. How could those parts of his square which were on one side of an edge come to the other side without the edge moving? He could understand their reflection in the edge. He could form an idea of the looking-glass image of his square lying on the opposite side of the line of an edge, but by no motion that he knows of can he make the actual square assume that position. The result of the rotation would be like reflection in the edge, but it would be a physical impossibility to produce it in the plane.

The demonstration of rotation about a line must be to him purely formal. If he conceived the notion of a cube stretching out in an unknown direction away from his plane, then he can see the base of it, his square in the plane, rotating round a point. He can likewise apprehend that every parallel section taken at successive intervals in the unknown direction rotates in like manner round a point. Thus he

[18] It is hard, in the light of what Hinton says in *A New Era of Thought*, to believe that he really means this—except in the sense where "visualize" is taken to mean "form a visual image" rather than "form a mental image." Given the basically formal nature of this paper, he undoubtedly did not want to seem too eager to convince his listeners of the living reality of the fourth dimension. [R.v.B.R.]

would come to conclude that the whole body rotates round a line—the line consisting of the succession of points round which the plane sections rotate. Thus, given three axes, x, y, z; if x rotates to take the place of y, and y turns so as to point to negative x, then the third axis remaining unaffected by this turning is the axis about which the rotation takes place. This, then, would have to be his criterion of the axis of a rotation—that which remains unchanged when a rotation of every plane section of a body takes place.

There is another way in which a plane being can think about three-dimensional movements; and, as it affords the type by which we can most conveniently think about four-dimensional movements, it will be no loss of time to consider it in detail.

We can represent the plane being and his object by figures cut out of paper, which slip on a smooth surface. The thickness of these bodies must be taken as so minute that their extension in the third dimension escapes the observation of the plane being, and he thinks about them as if they were mathematical plane figures in a plane instead of being material bodies capable of moving on a plane surface. Let Ax, Ay be two axes and $ABCD$ a square (fig. 45). As far as movements in the plane are concerned, the square can rotate about a point, A, for example. It cannot rotate about a side such as AC.

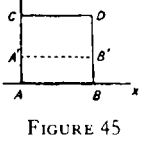


FIGURE 45

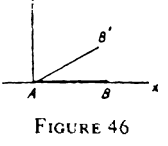


FIGURE 46

But if the plane being is aware of the existence of a third dimension he can study the movements possible in the ample space, taking his figure portion by portion.

His plane can only hold two axes. But, since it can hold two, he is able to represent a turning into the third dimension if he neglect one of his axes and represent the third axis as lying in his plane. He can make a drawing in his plane of what stands up perpendicularly from his plane. Let Az be the axis, which stands perpendicular to his plane at A. He can draw in his plane two lines to represent the two axes, Ax and Az. Let figure 46 be this drawing. Here the z axis has taken the place of the y axis, and the plane of $A\ x\ A\ z$ is represented in his plane. In this figure all that exists of the square $ABCD$ will be the line AB.

The square extends from this line in the y direction, but more of that direction is represented in figure 46. The plane being can study the turning of the line AB in this diagram. It is simply a case of plane turning around the point A. The line AB occupies intermediate portions like AB' and after half a revolution will lie on Ax produced through A.

Now, in the same way, the plane being can take another point, A', and another line, $A'B''$, in his square. He can make the drawing of the two directions at A', one along $A'B''$, the other perpendicular to his plane. He will obtain a figure precisely similar to figure 46, and will see that, as AB can turn around A, so $A'B''$ around A'.

In this turning AB and $A'B''$ would not interface with each other, as they would if they moved in the plane around the separate points A and A'.

Hence the plane being would conclude that a rotation round a line was possible. He could see his square as it began to make this turning. He could see it half way round when it came to lie on the opposite side of the line AC. But in intermediate portions he could not see it, for it runs out of the plane.

Coming now to the question of a four-dimensional body, let us conceive of it as a series of cubic sections, the first in our space, the rest at intervals, stretching away from our space in the unknown direction.

We must not think of a four-dimensional body as formed by moving a three-dimensional body in any direction which we can see.

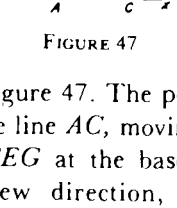


FIGURE 47

Refer for a moment to figure 47. The point A, moving to the right, traces out the line AC. The line AC, moving away in a new direction, traces out the square $ACEG$ at the base of the cube. The square $AEGC$, moving in a new direction, will trace out the cube $ACEGBDHF$. The vertical direction of this last motion is not identical with any motion possible in the plane of the base of the cube. It is an entirely new direction, at right angles to every line that can be drawn in the base. To trace out a tesseract the cube must move in a

new direction—a direction at right angles to any and every line that can be drawn in the space of the cube.

The cubic sections of the tesseract are related to the cube we see, as the square sections of the cube are related to the square of its base which a plane being sees.

Let us imagine the cube in our space, which is the base of a tesseract, to turn about one of its edges. The rotation will carry the whole body with it, and each of the cubic sections will rotate. The axis we see in our space will remain unchanged, and likewise the series of axes parallel to it, about which each of the parallel cubic sections rotates. The assemblage of all of these is a plane.

Hence in four dimensions a body rotates about a plane. There is no such thing as rotation round an axis.

We may regard the rotation from a different point of view. Consider four independent axes each at right angles to all the others, drawn in a four-dimensional body. Of these four axes we can see any three. The fourth extends normal to our space.

Rotation is the turning of one axis into a second, and the second turning to take the place of the negative of the first. It involves two axes. Thus, in this rotation of a four-dimensional body, two axes change and two remain at rest. Four-dimensional rotation is therefore a turning about a plane.

As in the case of a plane being the result of rotation about a line could appear as the production of a looking-glass image of the original object on the other side of the line, so to us the result of a four-dimensional rotation would appear like the production of a looking-glass image of a body on the other side of a plane. The plane would be the axis of the rotation, and the path of the body between its two appearances would be unimaginable in three-dimensional space.

Let us now apply the method by which a plane being could examine the nature of rotation about a line in our examination of rotation about a plane. Figure 47 represents a cube in our space, the three axes x, y, z, denoting its three dimensions. Let w represent the fourth dimension. Now, since in our space we can represent any three dimensions, we can, if we choose, make a representation of what is in the space determined by three axes x, z, w. This is a three-dimensional space determined by two of the axes we have drawn, x and z, and in place of y the fourth axis, w. We cannot, keeping x and z, have both y and w in our space; so we will let y go and draw w in its place. What will be our view of the cube?

Evidently we shall have simply the square that is in the plane of xz, the square $ACDB$. The rest of the cube stretches in the y direction, and, as we have none of the space so determined, we have only the face of the cube. This a represented in figure 48.

Now, suppose the whole cube to be turned from the x to the w direction. Conformably with our method, we will not take the whole of the cube into consideration at once, but will begin with the face $ABCD$.

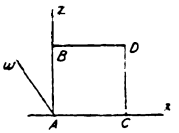


FIGURE 48

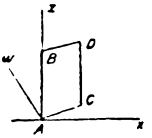


FIGURE 49

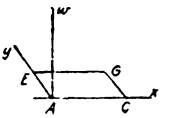


FIGURE 50

Let this face begin to turn. Figure 49 represents one of the positions it will occupy; the line AB remains on the z axis. The rest of the face extends between the x and the w direction.

Now, since we can take any three axes, let us look at what lies in the space of zyw, and examine the turning there. We must now let the z axis disappear and let the w axis run in the direction in which z ran.

Making this representation, what do we see of the cube? Obviously we see only the lower face. The rest of the cube lies in the space of xyz. In the space of xyw we have merely the base of the cube lying in the plane of xy, as shown in figure 50.

Now let the x to w turning take place. The square $ACEG$ will turn about the line AE (fig. 51). This edge will remain along the y axis and will be stationary, however far the square turns.

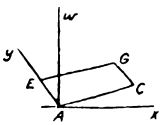


FIGURE 51

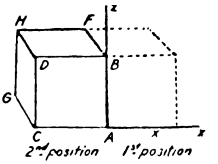


FIGURE 52

Thus, if the cube be turned by an x to w turning, both the edge AB and the edge AC remain stationary; hence the whole face $ABEF$ in the yz plane remains fixed. The turning has taken place about the face $ABEF$.

Suppose this turning to continue till AC runs to the left from A. The cube will occupy the position shown in figure 52. This is the looking-glass image of the cube in figure 47. By no rotation in three-dimensional space can the cube be brought from the position in figure 47 to that shown in figure 52.

We can think of this turning as a turning of the face $ABCD$ about AB, and a turning of each section parallel to $ABCD$ round the vertical line in which it intersects the face $ABEF$, the space in which the turning takes place being a different one from that in which the cube lies.

One of the conditions, then, of our inquiry in the direction of the infinitely small is that we form the conception of a rotation about a plane. The production of a body in a state in which it presents the appearance of a looking-glass image of its former state is the criterion for a four-dimensional rotation.

There is some evidence for the occurrence of such transformations of bodies in the change of bodies from those which produce a right-handed polarization of light to those which produce a left-handed polarization; but this is not a point to which any very great importance can be attached.

Still, in this connection, let me quote a remark from Prof. John G. McKendrick's address on physiology before the British Association at Glasgow. Discussing the possibility of the hereditary production of characteristics through the material structure of the ovum, he estimates that in it there exist 12,000,000,000 biophors, or ultimate particles of living matter, a sufficient number to account for hereditary transmission, and observes: "Thus it is conceivable that vital activities may also be determined by the kind of motion that takes place in the molecules of that which we speak of as living matter. It may be different in kind from some of the motions known to physicists, and it is conceivable that life may be the transmission to dead matter, the molecules of which have already a special kind of motion of a form of motion *sui generis*."[19]

Now, in the realm of organic beings symmetrical structures—those with a right and left symmetry—are everywhere in evidence. Granted

[19] The idea here seems to be that each gene is continuously rotating in the fourth dimension and thus turning into its mirror image and back. The vitalist belief that life must involve some unusual physics is now pretty well discredited; and the 4-D rotation idea is definitely wrong, since it has been determined that our genetic material is in the form of a left-handed DNA helix which does not ever appear in the right-handed form. [R.v.B.R.]

that four dimensions exist, the simplest turning produces the mirror-image form, and by a folding over, structures could be produced, duplicated right and left, just as in the case of a plane. A symmetrical and lifelike contour is created by the child's amusement of folding an ink-spattered paper along the line of blots.

Whether four-dimensional motions correspond to the physiologist's demand for a special kind of motion or not, I do not know. Our business is with the evidence for its existence in physics. For this purpose it is necessary to examine into the significance of rotation round a plane in the case of extensible and of fluid matter.

Let us dwell a moment longer on the rotation of a rigid body. Looking at the cube in figure 47, which turns about the face of *ABFE*, we see that any line in the face can take the place of the vertical and horizontal lines we have examined. Take the diagonal line *AF* and the section through it to *GH*. The portions of matter which were on one side of *AF* in this section in figure 47 are on the opposite side of it in figure 52. They have gone round the line *AF*. Thus the rotation round a face can be considered as a number of rotations of sections round parallel lines in it.

The turning about two different lines is impossible in three-dimensional space. To take another illustration (fig. 53), suppose *A* and *B* are two parallel lines in the *xy* plane, and let *CD* and *EF* be two rods crossing them. Now, in the space of *xyz* if the rods turn round the lines *A* and *B* in the same direction they will make two independent circles.

When the end *F* is going down the end *C* will be coming up. They will meet and conflict.

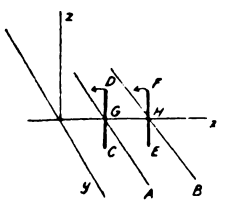


FIGURE 53

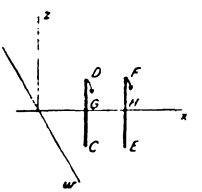


FIGURE 54

But if we rotate the rods about the plane of *AB* by the *z* to *w* rotation (fig. 54), these movements will not conflict. Suppose all the figure removed with the exception of the plane *xz*, and from this plane draw the axis of *w*, so that we are looking at the space of *xzw*.

Here, figure 54, we cannot see the lines *A* and *B*. We see the points

G and H, in which A and B intercept the x axis, but we cannot see the lines themselves, for they run in the y direction, and that is not in our drawing.

Now, if the rods move with the z to w rotation, they will turn in parallel planes, keeping their relative positions. The point D, for instance, will describe a circle. At one time it will be above the line A, at another time below it. Hence it rotates round A.

Now only two rods, but any number of rods crossing the plane will move round it harmoniously. We can think of this rotation by supposing the rods standing up from one line to move round that line and remembering that it is not inconsistent with this rotation for the rods standing up along another line also to move round it, the relative positions of all the rods being preserved. Now, if the rods are thick together, they may represent a disk of matter, and we see that a disk of matter can rotate round a central plane.

Rotation round a plane is exactly analogous to rotation round an axis in three dimensions. If we want a rod to turn round, the ends must be free; so if we want a disk of matter to turn round its central plane by a four-dimensional turning, all the contour must be free. The whole contour corresponds to the ends of the rod. Each point of the contour can be looked on as the extremity of an axis in the body, round each point of which there is a rotation of the matter in the disk.

If the one end of a rod be clamped, we can twist the rod, but not turn it round; so if any part of the contour of a disk is clamped we can impart a twist to the disk, but not turn it round its central plane. In the case of extensible materials a long, thin rod will twist round its axis, even when the axis is curved; as, for instance, in the case of a ring of India rubber.

In an analogous manner, in four dimensions we can have rotation round a curved plane, if I may use the expression. A sphere can be turned inside out in four dimensions.

Let figure 55 represent a spherical surface on each side of which a

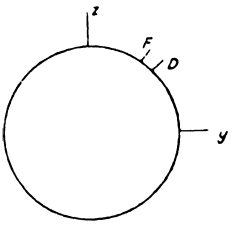


FIGURE 55

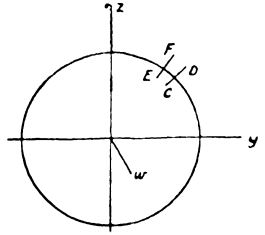


FIGURE 56

layer of matter exists. The thickness of the matter is represented by the rods CD and EF, extending equally without and within.

Now, take the section of the sphere by the yz plane; we have a circle—figure 56. Now, let the w axis be drawn in place of the x axis so that we have the space of yzw represented. In this space all that there will be seen of the sphere is the circle drawn.

Here we see that there is no obstacle to prevent the rods turning round. If the matter is so elastic that it will give enough for the particles at E and C to be separated as they are at F and D, they can rotate round to the position D and F, and a similar motion is possible for all other particles. There is no matter or obstacle to prevent them from moving out in the w direction, and then on round the circumference as an axis. Now, what will hold for one section will hold for all, as the fourth dimension is at right angles to all the sections which can be made of the sphere.

We have supposed the matter of which the sphere is composed to be three dimensional. If the matter had a small thickness in the fourth dimension, there would be a slight thickness in figure 56 above the plane of the paper—a thickness equal to the thickness of the matter in the fourth dimension. The rods would have to be replaced by thin slabs. But this would make no difference as to the possibility of the rotation. This motion is discussed by Newcomb in the first volume of the *American Journal of Mathematics*.

Let us now consider, not a merely extensible body, but a liquid one. A mass of rotating liquid, a whirl, eddy, or vortex, has many remarkable properties. On first consideration we should expect a rotating mass of liquid immediately to spread off and lose itself in the surrounding liquid. The water flies off a wheel whirled round, and we should expect the rotating liquid to be dispersed. But we see the eddies in a river strangely persistent. The rings that occur in puffs of smoke and last so long are whirls or vortices curved round so that their opposite ends join together. A cyclone will travel over great distances.

Helmholtz was the first to investigate the properties of vortices. He studied them as they would occur in a perfect fluid—that is, one without friction of one moving portion or another. In such a medium vortices would be indestructible. They would go on forever, altering their shape, but consisting always of the same portion of the fluid. But a straight vortex could not exist surrounded entirely by the fluid. The ends of a vortex must reach to some boundary inside or outside the fluid.

A vortex which is bent round so that its opposite ends join is capable of existing, but no vortex has a free end in the fluid. The fluid round a vortex is always in motion, and one produces a definite movement in another.

Lord Kelvin has proposed the hypothesis that portions of a fluid segregated in vortices account for the origin of matter. The properties of the aether in respect of its capacity of propagating disturbances can be explained by the assumption of vortices in it instead of by a property of rigidity. It is difficult to conceive, however, of any arrangement of the vortex rings and endless vortex filaments in the aether.[20]

Now, the further consideration of four-dimensional rotations shows the existence of a kind of vortex which would make an aether filled with a homogeneous vortex motion easily thinkable.

To understand the nature of this vortex, we must go on and take a step by which we accept the full significance of the four-dimensional hypothesis. Granted four-dimensional axes, we have seen that a rotation of one into another leaves two unaltered, and these two form the axial plane about which the rotation takes place. But what about these two? Do they necessarily remain motionless? There is nothing to prevent a rotation of these two, one into the other, taking place concurrently with the first rotation. This possibility of a double rotation deserves the most careful attention, for it is the kind of movement which is distinctively typical of four dimensions.

Rotation round a plane is analogous to rotation round an axis. But in three-dimensional space there is no motion analogous to the double rotation, in which, while axis 1 changes into axis 2, axis 3 changes into axis 4.

Consider a four-dimensional body, with four independent axes, x,y,z,w. A point in it can move in only one direction at a given moment. If the body has a velocity of rotation by which the x axis changes into the y axis and all parallel sections move in a similar manner, then the point will describe a circle. If, now, in addition to the rotation by which the x axis changes into the y axis the body has a rotation by which the z axis turns into the w axis, the point in question will have a double motion in consequence of the two turnings. The motions will compound, and the point will describe a circle, but

[20] If, however, we think of the aether as a thin hypersheet which we live inside of, then we *can* have vortex filaments which run from the top of the aether to the bottom. Or again, if we think of the aether as a sheet, then we can pass to Clifford's space theory of matter which proposes that particles are simply little hills in the aether. [R.v.B.R.]

not the same circle which it would describe in virtue of either rotation separately.

We know that if a body in three-dimensional space is given two movements of rotation, they will combine into a single movement of rotation round a definite axis. It is in no different condition from that in which it is subjected to one movement of rotation. The direction of the axis changes; that is all. The same is not true about a four-dimensional body. The two rotations x to y and z to w are independent. A body subject to the two is in a totally different condition to that which it is in when subject to one only. When subject to a rotation such as that of x to y, a whole plane in the body, as we have seen, is stationary. When subject to the double rotation no part of the body is stationary except the point common to the two planes of rotation.

If the two rotations are equal in velocity, every point in the body describes a circle. All points equally distant from the stationary point describe circles of equal size.

We can represent a four-dimensional sphere by means of two diagrams, in one of which we take the three axes x, y, and z; in the other the axes x,w, and z. In figure 57 we have the view of a four-dimensional sphere in the space of xyz. Figure 57 shows all that we can see of the four sphere in the space of xyz, for it represents all the points in that space, which are at an equal distance from the center.

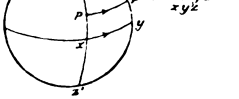


FIGURE 57

FIGURE 58

Let us now take the xz section, and let the axis of w take the place of the y axis. Here, in figure 58, we have the space of xzw. In this space we have to take all the points which are at the same distance from the center, consequently we have another sphere. If we had a three-dimensional sphere, as has been shown before, we should have merely a circle in the xzw space, the xz circle seen in the space of xzw. But now, taking the view in the space of xzw, we have a sphere in that space also. In a similar manner, whichever set of three axes we take, we obtain a sphere.

In figure 57, let us imagine the rotation in the direction xy to be taking place. The point x will turn to y, and p to p'. The axis zz'

remains stationary, and this axis is all of the plane zw which we can see in the space section exhibited in the figure.

In figure 58, imagine the rotation from z to w to be taking place. The w axis now occupies the position previously occupied by the y axis. This does not mean that the w axis can coincide with the y axis. It indicates that we are looking at the four-dimensional sphere from a different point of view. Any three-space view will show us three axes, and in figure 58 we are looking at xzw.

The only part that is identical in the two diagrams is the circle of the x and z axes, which axes are contained in both diagrams. Thus the plane z, x, z' is the same in both, and the point p represents the same point in both diagrams. Now, in figure 58 let the zw rotation take place, the z axis will turn toward the point w of the w axis, and the point p will move in a circle about the point x.

Thus in figure 57 the point p moves in a circle parallel to the xy plane; in figure 58 it moves in a circle parallel to the zw plane, indicated by the arrow.

Now, suppose both of these independent rotations compounded; the point p will move in a circle, but this circle will coincide with neither of the circles in which either one of the rotations will take it. The circle the point p will move in will depend on its position on the surface of the four sphere.

In this double rotation, possible in four-dimensional space, there is a kind of movement totally unlike any with which we are familiar in three-dimensional space. It is a requisite preliminary to the discussion of the behavior of the small particles of matter, with a view to determining whether they show the characteristics of four-dimensional movements, to become familiar with the main characteristics of this double rotation. And here I must rely on a formal and logical assent rather than on the intuitive apprehension which can only be obtained by a more detailed study.

In the first place this double rotation consists in two varieties or kinds, which we will call the A and B kinds. Consider four axes, x, y, z, w. The rotation of x to y can be accompanied with the rotation of z to w. Call this the A kind.

But also the rotation of x to y can be accompanied by the rotation of not z to w, but w to z. Call this the B kind.

They differ in only one of the component rotations. One is not the negative of the other. It is the semi-negative. The opposite of an x to

y, z to w rotation would be y to x, w to z. The semi-negative is x to y and w to z.

If four dimensions exist and we cannot perceive them because the extension of matter is so small in the fourth dimension that all movements are withheld from direct observation except those which are three dimensional, we should not observe these double rotations, but only the effects of them in three-dimensional movements of the type with which we are familiar.

If matter in its small particles is four dimensional we should expect this double rotation to be a universal characteristic of the atoms and molecules, for no portion of matter is at rest. The consequences of this corpuscular motion can be perceived, but only under the form of ordinary rotation or displacement. Thus if the theory of four dimensions is true we have in the corpuscles of matter a whole world of movement which we can never study directly, but only by means of inference.

The rotation A, as I have defined it, consists of two equal rotations—one about the plane of zw, the other about the plane of xy. It is evident that these rotations are not necessarily equal. A body may be moving with a double rotation in which these two independent components are not equal; but in such a case we can consider the body to be moving with a composite rotation—a rotation of the A or B kind and, in addition, a rotation about a plane.

If we combine an A and a B movement, we obtain a rotation about a plane; for, the first being x to y and z to w, and the second being x to y and w to z, when they are put together the z to w and w to z rotations neutralize each other, and we obtain an x to y rotation only, which is a rotation about the plane of zw. Similarly, if we take a B rotation, y to x and z to w, we get, on combining this with the A rotation, a rotation of z to w about the xy plane. In this case the plane of rotation is in the three-dimensional space of xyz, and we have what has been described before—a twisting about a plane in our space.

Consider now a portion of a perfect liquid having an A motion. It can be proven that it possesses the properties of a vortex. It forms a permanent individuality—a separated-out portion of the liquid accompanied by a motion of the surrounding liquid. It has properties analogous to those of a vortex filament. But it is not necessary for its existence that its ends should reach the boundary of the liquid. It is self-contained and, unless disturbed, is circular in every section.

If we suppose the aether to have its properties of transmitting vibra-

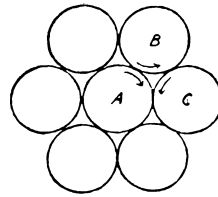


FIGURE 59

tion given it by such vortices, we must inquire how they lie together in four-dimensional space. Placing a circular disk on a plane and surrounding it by six others (fig. 59), we find that if the central one is given a motion of rotation, it imparts to the others a rotation which is antagonistic in every two adjacent ones. If *A* goes round as shown by the arrow, *B* and *C* will be moving in opposite ways, and each tends to destroy the motion of the other.

Now, if we suppose spheres to be arranged in a corresponding manner in three-dimensional space, they will be grouped in figures which are for three-dimensional space what hexagons are for plane space. If a number of spheres of soft clay be pressed together, so as to fill up the interstices, each will assume the form of a 14-sided figure, called a tetrakaidecagon.

Now, assuming space to be filled with such tetrakaidecagons and placing a sphere in each, it will be found that one sphere is touched by six others. The remaining eight spheres of the fourteen which surround the central one will not touch it, but will touch three of those in contact with it. Hence if the central sphere rotates it will not necessarily drive those around it so that their motions will be antagonistic to each other, but the velocities will not arrange themselves in a systematic manner.

In four-dimensional space the figure which forms the next term of the series hexagon, tetrakaidecagon, is a thirty-sided figure. It has for its faces ten solid tetrakaidecagons and twenty hexagonal prisms. Such figures will exactly fill four-dimensional space, five of them meeting at every point. If, now, in each of these figures we suppose a solid four-dimensional sphere to be placed, any one sphere is surrounded by thirty others. Of these it touches ten, and, if it rotates, it drives the rest by the means of these. Now, it we imagine the central sphere to be given an A or a B rotation, it will turn the whole mass of spheres round in a systematic manner. Suppose four-dimensional space to be

filled with such spheres, each rotating with a double rotation, the whole mass would form one consistent system of motion, in which each one drove every other one, with no friction or lagging behind.

Every sphere would have the same kind of rotation. In three-dimensional space, if one body drives another round, the second body rotates with the opposite kind of rotation; but in four-dimensional space these four-dimensional spheres would each have the double negative of the rotation of the one next it, and we have seen that the double negative of an A or B rotation is still an A or B rotation. Thus four-dimensional space could be filled with a system of self-preservative living energy. If we imagine the four-dimensional spheres to be of liquid and not of solid matter, then, even if the liquid were not quite perfect and there were a slight retarding effect of one vortex on another, the system would still maintain itself.

In this hypothesis we must look on the aether as possessing energy, and its transmission of vibrations, not as the conveying of a motion imparted from without, but as a modification of its own motion.

We are now in possession of some of the conceptions of four-dimensional mechanics, and will turn aside from the line of their development to inquire if there is any evidence of their applicability to the processes of nature.

Is there any mode of motion in the region of the minute which, giving three-dimensional movements for its effect, still in itself escapes the grasp of our mechanical theories? I would point to electricity. Through the labors of Faraday and Maxwell we are convinced that the phenomena of electricity are of the nature of the stress and strain of a medium; but there is still a gap to be bridged over in their explanation—the laws of elasticity, which Maxwell assumes, are not those of ordinary matter. And, to take another instance: a magnetic pole in the neighborhood of a current tends to move. Maxwell has shown that the pressures on it are analogous to the velocities in a liquid which would exist if a vortex took the place of the electric current; but we cannot point out the definite mechanical explanation of these pressures. There must be some mode of motion of a body or of the medium in virtue of which a body is said to be electrified.

Take the ions which convey charges of electricity 500 times greater in proportion to their mass than are carried by the molecules of hydrogen in electrolysis. In respect of what motion can these ions be said to be electrified? It can be shown that the energy they possess is not energy of rotation. Think of a short rod rotating. If it is turned over it

is found to be rotating in the opposite direction. Now, if rotation in one direction corresponds to positive electricity, rotation in the opposite direction corresponds to negative electricity, and the smallest electrified particles would have their charges reversed by being turned over—an absurd supposition.

If we fix on a mode of motion as a definition of electricity, we must have two varieties of it, one for positive and one for negative; and a body possessing the one kind must not become possessed of the other by any change in its position.

All three-dimensional motions are compounded of rotations and translations, and none of them satisfy this first condition for serving as a definition of electricity.

But consider the double rotation of the A and B kinds. A body rotating with the A motion cannot have its motion transformed into the B kind by being turned over in any way. Suppose a body has the rotation x to y and z to w. Turning it about the xy plane, we reverse the direction of the motion x to y. But we also reverse the z to w motion, for the point at the extremity of the positive z axis is now at the extremity of the negative z axis, and since we have not interfered with its motion, it goes in the direction of position w. Hence we have y to x and w to z, which is the same as x to y and z to w. Thus both components are reversed, and there is the A motion over again. The B kind is the semi-negative, with only one component reversed.

Hence a system of molecules with the A motion would not destroy it in one another, and would impart it to a body in contact with them. Thus A and B motions possess the first requisite which must be demanded in any mode of motion representative of electricity.

Let us trace out the consequences of defining positive electricity as an A motion and negative electricity as a B motion. The combination of positive and negative electricity produces a current. Imagine a vortex in the aether of the A kind and unite with this one of the B kind. An A motion and a B motion produce rotation round a plane, which is in the aether a vortex round an axial surface. It is a vortex of the kind we represent as a part of a sphere turning inside out. Now, such a vortex must have its rim on a boundary of the aether—on a body in the aether.

Let us suppose that a conductor is a body which has the property of serving as the terminal abutment of such a vortex. Then the conception we must form of a closed current is of a vortex sheet having its edge along the circuit of the conducting wire. The whole wire will

then be like the centers on which a spindle turns in three-dimensional space, and any interruption of the continuity of the wire will produce a tension in place of a continuous revolution.

As the direction of the rotation of the vortex is from a three-space direction into the fourth dimension and back again, there will be no direction of flow to the current; but it will have two sides, according to whether z goes to w or z goes to negative w.

We can draw any line from one part of the circuit to another; then the aether along that line is rotating round its points.

This geometric image corresponds to the definition of an electric circuit. It is known that the action does not lie in the wire, but in the medium, and it is known that there is no direction of flow in the wire.

No explanation has been offered in three-dimensional mechanics of how an action can be impressed throughout a region and yet necessarily run itself out along a closed boundary, as is the case in an electric current. But this phenomenon corresponds exactly to the definition of a four-dimensional vortex.

If we take a very long magnet, so long that one of its poles is practically isolated, and put this pole in the vicinity of an electric circuit, we find that it moves.

Now, assuming for the sake of simplicity that the wire which determines the current is in the form of a circle, if we take a number of small magnets and place them all pointing in the same direction normal to the plane of the circle, so that they fill it and the wire binds them round, we find that this sheet of magnets has the same effect on the magnetic pole that the current has. The sheet of magnets may be curved, but the edge of it must coincide with the wire. The collection of magnets is then equivalent to the vortex sheet and an elementary magnet to a part of it. Thus, we must think of a magnet as conditioning a rotation in the aether round the plane which bisects at right angles the line joining its poles.

If a current is started in a circuit, we must imagine vortices like bowls turning themselves inside out, starting from the contour. In reaching a parallel circuit, if the vortex sheet were interrupted and joined momentarily to the second circuit by a free rim, the axis plane would lie between the two circuits, and a point on the second circuit opposite a point on the first would correspond to a point opposite to it on the first; hence we should expect a current in the opposite direction in the second circuit. Thus the phenomena of induction are not inconsistent with the hypothesis of a vortex about an axial plane.

In four-dimensional space in which all four dimensions were commensurable, the intensity of the action transmitted by the medium would vary inversely as the cube of the distance. Now, the action of a current on a magnetic pole varies inversely as the square of the distance; hence over measurable distances the extension of the aether in the fourth dimension cannot be assumed as other than small in comparison with those distances. This extension being small, the effect of a vortex sheet would be equivalent to a number of jets on one side and suctions on the other.

Such an arrangement in the case of a liquid would produce velocities in the liquid which coincide in direction with the tendency of motion of a magnetic pole. But analogies of this kind leave out of sight the fact that the action is a reciprocal one. Non-magnetic matter shows no tendency to move. To arrive at a definite conclusion it will be necessary to investigate the resultant pressures which accompany the collocation of solid vortices with surface ones.

To recapitulate: The movements and mechanics of four-dimensional space are definite and intelligible. A vortex with a surface as its axis affords a geometric image of a closed circuit, and there are rotations which by their polarity afford a possible definition of statical electricity.

AN EPISODE OF FLATLAND

[This selection contains excerpts of *An Episode of Flatland* (1907), including material from the Introduction, "The History of Astria," and Chapters 6, 7, 8, 9, 10 and 20.]

Introduction

Placing some coins on the table one day (fig. 60), I amused myself by pushing them about, and it struck me that one might represent a planetary system of a certain sort by their means. This large one in the center represents the sun, and the others its planets journeying round it. And in this case considering the planets as inhabited worlds, confined in all their movements round their sun, to a slipping over the surface of the table, I saw that we must think of the beings that inhabit these worlds as standing out from the rims of them, not walking over the flat surface of them. Just as attraction in the case of our earth acts towards the center, and the center is inaccessible by reason of the solidity on which

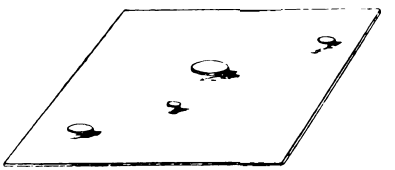


FIGURE 60

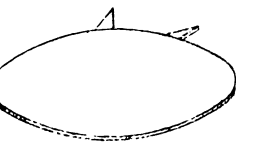


FIGURE 61

we stand, so the inhabitants of my coin worlds would have an attraction proceeding out in every direction along the surface of the table from the center of the coin, and "up" would be to them out from the center beyond the rim, while "down" would be towards the center inwards from the rim. And beings thus situated would be rightly described as standing on the rim (fig. 61).

And I saw that if I supposed the surface of the table to be perfectly smooth, so that there was no impediment to motion along it, then

these beings would have no notion at all that there was a surface on which they slipped. Since the surface is always in contact with every moving thing, the notion of it would be absent from their consciousness. There would be no difference in respect to it. And I saw that here I had an image of a two-dimensional world, a world in which the creatures of it would think that space itself was two dimensional.

We see that the discs which form these worlds must be supported somehow, but the beings of such a universe would not ask such a question—they would think that all the space there was lay in the extension of the movements they made, and would never think of any movement away from or into the table, being always in contact with it.

But it is very hard to realize how "out" from a disk, such as one of my coins, could be felt as "up" and inward towards the center of it would be felt as "down." To ease my mind on this point I imagined myself standing on the equator of our earth, looking along it, and then a great steel blade coming down and cutting the earth right through along the equator circle, and then coming down again and cutting a slice parallel to the first. And then I imagined this slice of the earth and myself sticking against the steel blade, like the slice of a pea against a knife blade. In this way I gained the feeling of a being on a disk, with an "up and down," "away from and to" the center of the disk.

But still I had a consciousness of another direction than those of "forward and backward"—along the rim of the disk—and "up and down" away from its center and towards it. I could not help predicating myself with the sense of right and left—away from and into the steel blade. To lose this sense I must evidently change my notion of the constitution of my body. Without carrying the cutting so far as to imagine myself sliced, I imagined myself as made of very thin material, just of the width of the slice of the earth, and supposed that I myself and all the matter of the slice were of the same thickness, and stood out from the blade to exactly the same amount.

"If now," I said, "I was unconscious of this thickness, if the blade was perfectly smooth, and I and all the matter I knew slipped perfectly freely over it, I should have a two-dimensional experience. My arm in moving, or my finger in pointing, could only move in contact with the blade, and I could never point in a third dimension. I should not think of it, for all motions of all things take place along the surface of the blade."

Thus it became apparent that without making the supposition that I

was a mere line or triangle, or other geometrical figure, I could imagine myself as a two-dimensional being. If my thickness were very small and I was unconscious of it, if I could never move away from contact with a surface, my experience would be that of a two-dimensional being. So, after all, it seemed possible that there could be real two-dimensional beings. Now, if a thing is real, the only reason for not seeing it is either that it is small, or very far away—or some other reason. Hence I began to set about to try to discover these two-dimensional beings, and learn all about them. I succeeded at last, and if I do not tell you how, I am afraid it is from no very worthy motive. For if I told how one could find out about them, I am afraid Mr. Wells or Mr. Gelett Burgess[21] or some other brilliant author would begin to write about them, and to serve them up with all the resources of wit and humor. In that case no one would listen to me. As it is I intend to have the pleasure of telling about them myself.

One thing always puzzled me from the beginning of the time when I began to think about these plane beings, and that is about their eyes.

It is clear that they could not have two eyes beside one another as we have, for there is not the thickness in their bodies to place them so. Now, if they had two eyes, I wondered if one was above the other, or if they had one eye in front, another in the back of their heads. About this and other questions, I gained subsequently all the information one can desire. I have come to think of these creatures, from what I have found out, as very like ourselves, in different physical conditions it is true—but motives, aims and character, vary but little, however conditions differ. The only broad characteristic of difference I would draw, is that they are not so massive as we. They are more easily moved to action, and political and other changes are brought to pass more easily than with us. They also take narrower views than we do, they do not look on things in the broad and tolerant way we do.

In order to place before the reader all that I have to say systematically, I will begin with a short history of Astria, summarizing the events which took place on that planet from the earliest times till I come to the epoch about which I have written in more detail. With regard to that period, by selecting from the materials at my disposal, I have given some definite and personal information about characters who played an important part in late events.

[21] Gelett Burgess was the friend of Hinton's who wrote his obituary. He had some reputation as a humorist, and was the author of a great ghost story called "The Ghost Extinguisher" which appeared in April 1905 in *Cosmopolitan*. [R.v.B.R]

Astria is a plane world, along the rim of which its inhabitants walk. "Up" is away from the center of the disk, "down" is towards the center. To save myself the trouble of going into anatomical details, I will represent an Astrian, diagrammatically, by means of a triangle. And it will be conducive to the clearness of the reader's imagination, if he will suppose the great sheet of matter against the surface of which Astria, its sun, and all the material bodies of that universe slip, to be disposed vertically. He will then gain a more real presentment of the feelings of motion and progression in this world.

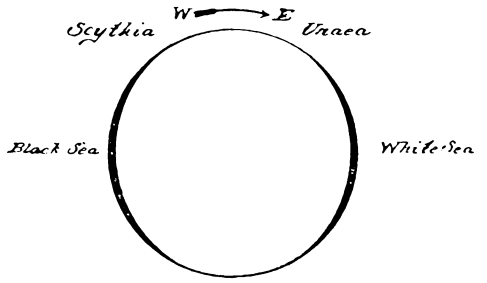


FIGURE 62

The edge of the plane world of Astria is divided into two approximately equal portions by two oceans—the Black Sea and the White Sea (fig. 62). Since the daily motion of rotation of Astria takes place in the direction denoted by an arrow, the sun appears to rise over the White Sea, and the direction from the inhabited region to the White Sea is called "east."

In the earliest times the inhabited region was divided amongst two peoples, the Unaeans and the Scythians. Of these the Unaeans were by far the more civilized. In fact, all that gave Astria the promise of becoming the gem of her planetary system, was to be found amongst the Unaeans, while the Scythians led a predatory nomadic existence. Yet, versed as they were in all the arts of life, the Unaeans, from the dawn of history, were gradually forced back and conquered by the Scythians.

Caesar in his history of the wars in Gaul, speaking of the provincials, says that it was their culture that made them fall before the barbaric and hardy valor of the Germans. He speaks as if civilization and culture brought of itself something debilitating and weakening to the

sterner virtues. But a different reason must be assigned for the constant defeats of the Unaeans, the constant ravishment of their territory, the continual absorption of a region of light into darkness by the Scythian hordes, who spared neither age nor sex, and never relinquished the land they once had gained.

I will explain the cause of the Unaeans' ill success. My rough and ready representation of the inhabitants of Astria, by means of a triangle (fig. 63), is sufficient to enable me to describe the main features of their bodily configuration.

This figure of a triangle I use in a conventional way as a mark or symbol, which is simple and easy to draw, and which without any un-

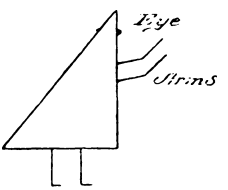


FIGURE 63

necessary complication enables me to make matters plain. It shows a thing I have often wondered at, namely, that there is a certain indication in the Astrian frame of being fashioned after the pattern of a higher existence rather than of complete adaptation to the exigencies of its narrow world.

Looking at the triangle which represents an Astrian, we see that on one edge are two arms and an eye, while on the other edge there exist no organs of sense or prehension. Thus, in going to the east, an Astrian could see his way clearly, and in working on anything, if it was placed to the east of him, he could operate on it conveniently; objects to the west, however, could only be seen by his bending over, and assuming a posture which, despite the suppleness of his frame, it was difficult to assume and painful to maintain for any length of time. Objects to the west also could only be reached at in a very awkward and ineffective manner.

It seems to us as if it would be an easy thing for an Astrian to turn round so that he could face in the west direction. But to do this we would have to lift the thin body of the man away from the sheet against which it slips. Such an operation is, of course, inconceivable to the inhabitants of a plane world, and their bodies would not stand such an operation, for they are far too thin to be safely turned about

and even temporarily deprived of the support of the sheet on which they slip. Every man in Astria was born facing the east, and facing the east he continued till he died.

Now I believe it is evident why the Scythians evidenced such a superiority over the Unaeans in warfare. The constitution of the Astrian body was such, that a Scythian man had an advantage over a Unaean man of a kind that no skill or discipline could countervail.

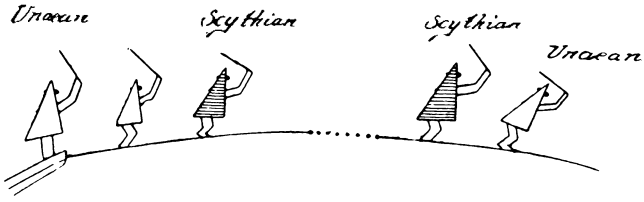


FIGURE 64

The Scythian whom I represent as a shaded figure (fig. 64), could both see the Unaean plainly and deliver blows at him to good effect, while the Unaean whom I represent as an unshaded figure, could only see the Scythian by a difficult exertion, and could only attack him or strike at him indirectly and backwards.

Thus year after year the Unaeans were forced within ever narrowing boundaries, till at last, with the White Sea on one side, their irresistible foes on the other, there seemed no outcome other than final and absolute extinction.

Yet with this hopeless prospect there was no demoralization of the national character: literature and art turned to motives of a more serious nature than in times when the danger was less closely realized, and the greatest minds devoted themselves to the inculcation of a brave and stoical courage, and a religion which robbed death of its terrors.

It is easy to say in the light of after events, that the intellectual energy of the race would have been better employed in the exploration of nature, and the wresting from its secrets of more efficacious methods of warfare. But the obvious is always found by an unobvious path. Unaean history affords no exception to the rule, as the following account of their discovery how to oppose the Scythians will show.

Amongst the men of this slowly perishing race were found some who withdrew their minds from all the fears of their time, and, with a detached and impersonal interest, studied the movements of the distant

stars. Thus with the Unaeans, as with us, in Astronomy, Science was born. Science, that interest, that appreciation of things in and for their own sake which we are accustomed to think of as the product of a prosperous and leisured community, sprang up in Unaea, when the bulwarks of their national existence were crumbling before the savage insistent blows of their inveterate enemy.

And as with us, Science in Astronomy gave its first gift to man, giving us the art of navigation, so in Unaea Science through Astronomy gave its first gift to these mortals. But the gift did not consist merely in the facilitation of an art. It was of unparalleled splendor, nothing less than the salvation of their race. For, studying the mutations of the heavenly bodies, accounting for their vicissitudes, eclipses and disturbances, astronomers came to the great thought of roundness of their earth. And as the news spread, as the tidings passed from one to the other that their earth was assuredly round, without any other words a great joy filled the hearts of this intelligent people. For everyone recognized without comment, that if their world was round, then a Unaean circumvading their disk would be in a position of as great advantage over a Scythian as the Scythians then had over them.

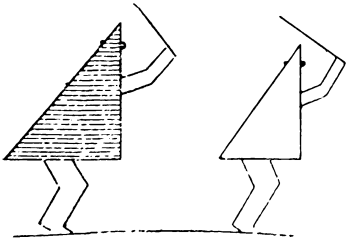


FIGURE 65

For we see that, on the right of the picture I give (fig. 65), we have the Scythian and Unaean in their ordinary position of combat; but on the left we see a Unaean who has circumvaded his globe, and comes on the Scythian in a position of advantage.

The prospect of meeting their hereditary foes under such reversed conditions, inspired the nation with the greatest ardor, and a period of astronomical discovery, equivalent to that which lies between the labors of Ptolemy and Newton was traversed in but few years. The Unaeans surmounted the difficulties of astronomical observation which were indeed considerable.

A tube, for instance, cannot be used in Astria—there is no means by which the opposite sides can be kept together. In order to observe the

transits of the heavenly bodies, it was necessary to make holes in the earth. The accompanying diagram (fig. 66) shows a Unaean telescope—a passage into the ground surmounted by a lens. It is obvious that the astronomer must descend to his place of observation by the same channel as that through which he takes his observations. If another opening were made, as shown in the second diagram, the earth above the chamber would fall into it, there being no support to keep it in its position.

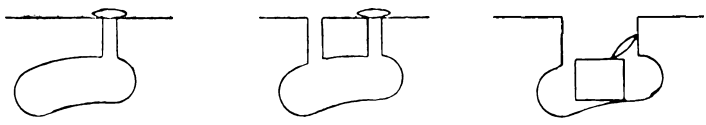


FIGURE 66

To overcome such difficulties which accompanied their mining operations, as well as their astronomical observations, the Unaeans applied all the resources of their active intelligence. And before many decades had passed, after the discovery of the circularity of the earth, they had gained enough information about the phenomena of tides to predict the existence of an antipodal continent, for the rise and fall of the sea on their shore was less than that which would take place if the White and Black Seas were two limits of the same ocean.

The existence of this continent rendered an expedition to take the Scythians in the rear practicable. But, though possible, it differed in its details from any other military operation ever put into execution.

The difficulties of traversing a virgin continent were in Astria almost insuperable. In the inhabited lands all the forests had been felled and the soil was covered in summer with a springy elastic cereal, which bore its fruit in a coiled up frond. Over the resilient surface afforded by this kind of vegetation, it was possible to travel with speed and comfort. But in the primeval forest the case was different.

It is obvious that of two Astrians meeting one would be compelled to climb over the other in order to pass him. We can imagine their condition by thinking of two tightrope walkers who, since they can diverge neither to the right or left, must pass one above or below the other. They would have the notion of right and left, although they could not make use of it; but the Astrians neither had the notion, nor if they had it could they have made use of it, all their movements being limited to such as could be executed under the conditions of their ma-

terial existence, that is, of not leaving the surface of the sheet against which they slipped. If merely to pass another individual afforded this measure of difficulty, it is conceivable how great an obstacle a single tree presented to progression. It had to be surmounted as it stood, or if cut down, then as it lay, with all its tangled mass of branches.

Taking the difficulties of navigation, of the penetration into the unknown continent, the emergence therefrom, and the building of ships to traverse the Black Sea, the lowest interval of time which could elapse between the departure of the expedition and the arrival of its survivors, was estimated at one hundred and fifty years. Of those that started none could reach the goal. A part of the nation must detach itself. A band of resolute hearts, brave and bold and faithful, must be chosen for such an enterprise.

Into the incredible wastes of the forlorn antipodes must pass a chosen band. For all their lives, and for their children's children's lives, they must traverse the labyrinthine branches of a primeval forest, with nothing of all Unaea held of gracious, fair and honorable shining on them. And yet they must keep their love for her; in the hearts of unborn generations the star of patriotism must rise, keeping them faithful in the weary way, where stage by stage they bore the burden of their country's last and only hope.

The expedition started, and all Unaea bent herself with a new spirit to the task of continuing their unequal fight. They even thought of training women fighters, a thing deemed incredible before.

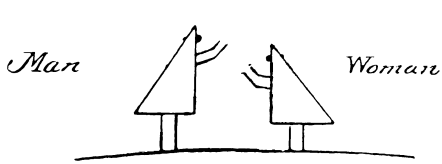


FIGURE 67

In speaking of the Astrians, I have previously only drawn men's figures, which are, as can be observed, all necessarily turned to the east. To represent a woman, however, it is necessary to draw a figure turned in the opposite direction, to the west (fig. 67).

Thus a Unaean woman, if her weakness and timidity were overcome by training, would be rightly framed to resist an attack from the west. The natural responsiveness of men to women and women to men, which we notice in our world, exists in Astria to a very highly accentuated degree. There a man cannot see his friend's face, because

it is necessarily turned from him, but he can watch a woman's face and note the changes of expression his words call forth. The Unaeans showed great chivalry in their treatment of women, and it was handed down as one of the most terrible horrors of that last period of their war, that actually a serious proposal had been entertained of women sacrificing womanhood, of hurling women into the contest against the Scythian oppressors.

How well the trust confided to the wanderers over land and sea was borne, is a theme Unaean writers love to linger on. From those that plunged in the waste arose a race lithe, adventurous, daring, with but one thought, the thought of fair Unaea. At the resting places for exercise in arms, at the nightly camps, at the halting places where some beetling crag affronted them, ever and again the old story was told, of Unaea fair and distant waiting for them, and none the less beautiful and clear was the story, because their words grew few and the dialect of these wood wanderers differed strangely from the sweet cadence of the Unaean tongue.

The sight of the crystal waters of the sea, when their hundred-year march was at an end, came to them like the fulfillment of a prophecy. They built their ships, traversed the ocean, and attacked their hereditary foes with an impact in which was concentrated, in a single impulse of destruction, all the energy, grace, thought and aspiration of their race.

Their onslaught was irresistible. When they reached the confines of their fatherland, Scythia, as a nation, had ceased to exist.

By the advent of these, her long separated children, the terrible oppression which had always weighed on Unaea was removed. All her most energetic sons had turned to war, now they turned to the arts of peace. And with the final and absolute disappearance of any power that could contest their might, a strange vicissitude of opinion took place. By the very perfection of its success the army had prepared the way for the extinction of the estimation in which it was held.

The survivors of the adventurous band, the old heroes who had held the Scythians at bay, were rewarded with gifts of land. Then Unaea settled down to think of other things than war. It fared but ill with the majority of the old-time fighters, they were ill-fitted for business, and the alluring schemes and skillful machinations of sharper men accomplished the ruin of many. That the army should have resigned itself and become a factor of no account, have passed from absolute do-

minion to become the merest cypher, was due to two causes. The general who had, by the force of his native capacity for command, come to the leadership of the invaders of Scythia, was a man named Wall, characterized by an absolute and simple devotion to his country. Wall's saying, "Soldier and servant," became the watchword of the solider men of the military class. So much for the personal cause. The other reason for the quiet disappearance of militarism was the wise provision of the capitalist class, who, foreseeing the time when the struggles with labor might become acute, established as part of the constitutional institutions of their land a standing army. The function of this army was essentially merely that of a highly-organized and very efficient gendarmerie, but by establishing a high rate of pay—for the class of work involved—and by the precaution of debarring from enlistment those who came from classes likely to be disaffected to the prevailing order of things, the governing class prepared a very real safeguard against sedition.

The wisdom shown by the founders of the modern institutions of Unaea was amply evidenced by the subsequent course of events.

After the first period of expansion was over, the versatile and enterprising genius of the people showed itself in a rapid course of organization, and the exploitation of every possible source of advantage for the organizations formed.

The working men had organizations which embraced all the skilled labor in every pursuit. The capitalists were united in organizations which controlled the supply of every kind of natural wealth. Between these two antagonistic bodies, the smaller employers disappeared. Labor and Capital were left face to face with each other, and on the side of Capital, with its traditions for private rights, government by the best, mastery and direction of the forces of the community, stood the army, a complete protection against any attempt to set aside the constitution by force.

Class distinctions came to be founded simply on wealth. The glamour of old times, when the preservation of the national existence rested on it, had completely departed from the army—save in the traditions which lived on in the army itself.

The mass of the people looked on the soldiers as mercenaries in the pay of the capitalists, while the capitalists regarded the soldiers as one amongst the various classes of men who were willing to work for them for a moderate rate of pay. Such is an epitome of Unaean history

preceding the period which I have made the subject of a special study.

[In the first few chapters we meet the star-crossed lovers, Laura Cartwright and Harold Wall. Laura's father is a powerful and unprincipled businessman who has come to control Unaea. Harold is a brave young soldier, son of the man who led the successful expedition against the Scythians. In order to prevent the marriage of Harold and Laura, Cartwright has talked Harold into going to settle a colony overseas. Heartbroken, Laura decides to go live with her eccentric uncle, Hugh Farmer, and study science with him.]

On Lone Mountain

Shortly after the fête of flowers, Laura started to pay a long visit to her uncle in Scythia. But before recounting the events which happened at Lone Mountain, a few words as to the character and life of Hugh Farmer will not be out of place. He was the only man in Unaea who believed in the third dimension. The occurrences by which Farmer was led to form his belief form an episode which is one of the most curious that can be conceived as happening in Flatland or any region of space. In Unaea, as with us, it was quite customary to represent a number by a line, and the square of a number by a square. If the number 2 was represented by a line, then 4, the square of that number, was represented by the square on that line. It was also quite obvious to the Unaeans that the cube of 2, or 8, could be represented by a figure with one more dimension than a square. They had the formal notion of a cube. But to conceive that such a figure actually existed, contradicted every principle of their science. For science involves a basis of observation—something given by the senses on which thought acts. On the other hand, to conceive a space of three dimensions meant giving thought a wrong part to play, a wrong function to perform. Thought would not give existence, it could only operate about existing things. The Unaean thinkers would as soon believe in chimeras or dragons as in three-dimensional space. For those notions, just as three-dimensional space, were deduced from thought, not founded on evidence of the senses. Farmer shared to the full this ardor of conviction of the supremacy of sense in giving materials for thought to work on. But in his day there was a sect who held the doctrine that the soul was separate and distinct from the body. They claimed that it

was possible to hold communion with the spirits of the dead, and they asserted that these ghostly beings could make themselves seen and felt. Now anything that could be seen and felt Farmer held to be a legitimate object of scientific investigation, so he took up the study of these spiritualistic phenomena. He fell a victim to jugglery. Tricks were played on him, and curious and inexplicable phenomena were produced in his presence. And it so happened, by a curious coincidence, that the marvellous occurrences which were palmed off on him were of a kind which could physically be produced if there were a third dimension. The tricks were wonderful enough in his space. But in three-dimensional space they would not be wonderful at all. For instance, in Flatland a box is a four-sided enclosure like a hollow square. Now Farmer saw objects outside such a four-sided enclosure, and afterwards inside it, without the sides being disturbed. Of course as a three-dimensional feat there would be no difficulty in transferring an object from the outside of a square to the inside. It would simply consist in taking it up and putting it down in a different place. Such occurrences and others Farmer witnessed. He believed they really happened. They made him believe in a three-dimensional space. He had that impact of the senses which was the only way a scientific and thoughtful Unaean could be got to believe anything. And this impact of the senses, this seeing and touching in the curious and roundabout way I have described made him believe in the third dimension. When the first step was taken, of course, and when he mentally habituated himself and became familiar with three-dimensional shapes, they afforded no more difficulty. They were evidently natural, and he saw it was absurd to limit existence to a plane. But his enthusiasm over his new conceptions led him into quarrels and disagreements with his contemporaries. He found it expedient to retire to a little property he owned in Scythia. There he shook off the load of other people's disapproval, and in solitary blessedness lived himself into the knowledge of three dimensions.

Nothing could have equalled Hugh Farmer's surprise and annoyance when his beautiful niece appeared and announced her intention of studying science with him. He told her he had absolutely no time and retreated into his inmost den. But she busied herself in his rooms, putting flowers on his shelves, and when hunger drove him out, he found it not so disagreeable to sit down opposite a fresh young face.

As is often the case with people who really know something, he was

the last kind of a man for a young person to go to in search of information. He began to think aloud.

"Have you never thought it strange," he said, "that there should be two shapes, each of which is exactly alike in its disposition of parts, but such that we can't turn one into another."

"There are not any shapes like that," she said.

"Yes there are," and he showed her two triangles* and pointed out that each had the same angles and the same lengths of sides, but one couldn't turn into the other, however much she moved them about.

Laura looked at them, and they reminded her of the little make-believe figures, the dolls she played with as a child, for those dolls were cut out in the form of triangles, and the triangle turned one way was always used for the boy doll, and the triangle turned the other way for the girl doll.

"They are like the dolls I used to make," she said, "one is like the boy doll and the other is like the girl doll."

"Yes," he said, "and could you ever turn one into the other?"

"No," she said.

"Why not?"

"Why should you want to?"

He groaned. "I didn't say I did want to, but if two things are exactly alike they ought to be able to be put in the same space."

"Of course they ought," she said, trying to please him.

"Well," he said, "if you think of a third dimension you could turn one into the other."

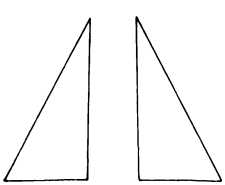
"Oh, I have heard of the third dimension," she said.

"Yes, what do you know about it?"

"It is where our souls go, our spirits, I mean; of course there you could turn one into the other. That's what they mean by saying that there is no difference between men and women in the land above—it's just the same for dolls as for us."

"I didn't know your father had an idiot for a daughter," said the old man, and went off to his den.

Laura rather liked it. It was so different from the way she was



* It is difficult to follow the old man's argument, but if one cuts out a couple of triangles like these (1 and 2), it is easy to see that keeping them against the surface of the paper they cannot be made to coincide.

generally spoken to. In fact, her uncle impressed her tremendously. And that he didn't like her, nerved her to do battle to win him. His books were all around. She took one to her bedroom, and failing to make any sense of the signs began to copy them out, and learn how to make curious marks of that kind. In the stillness of the night she frequently heard a tramp—up and down, up and down—and when she fell asleep it had not ceased—restless, nervous steps, as of some caged and suffering creature. She looked at her uncle nervously the next morning. He did not look any different, but she was sure he had hardly slept.

"Uncle," she said, "what is it makes you so unhappy?"

"Why do you think I am unhappy?"

"I know you are."

"I am an old man and must die soon."

"But all old men are not unhappy."

"Ah, but they leave the world young and fresh. Even before I go a wave of cold will strike us all, and lakes and seas will freeze. No green thing will blossom, only a few in deep caves, or, with soon-to-be-extinguished fires, will struggle on in a miserable existence that will be the end of the greatness of our earth."

"No, Uncle," she said, "you exaggerate. Papa told me something of this. He said the winters would be very bad; but I am sure it will not be so bad as you think."

"Child, your father mercifully hid his knowledge from you."

"Oh, Uncle, is that why he looked so worn and sad?"

"Yes. To bear a hopeless secret like that is enough to make him worn and sad. Only its absolute certainty could force him to admit it. He sent me the work he had done and I found that his calculators had taken the favorable supposition in every doubtful case. I do not blame them. The alternative is too terrible. On the most favorable supposition after the next meeting with that great planet, Ardaea, our earth will swing into a new orbit—we shall go far out into cold space till the earth is frozen deep, then we shall rush back so close to the sun that every day the surface of the earth will be seething hot. Perhaps some of us may maintain bare existence in deep caverns and hollows."

"If you are sure of this, Uncle, you ought to tell it that we may all prepare."

"Prepare for what?"

"Why, dear Uncle, our bodies are not all. If you or I die we know that our souls survive, and are judged by all the good and ill in our lives. You ought to tell everyone."

"And let them destroy all law and order in one short carousal? No, Laura, you do not know the world. There is more common sense in the common people than in all your bigoted idealists. We are here for a work and not for a theatrical play to manifest good qualities, and if men know that this work is to come to an end the fallacies of their preachers won't have any effect on them."

"They are not fallacies, Uncle."

"They are worse, Laura. They are interested deceptions. It grinds my heart out, child, to hear those glib preachers showing the way so confidently on evidence which no ordinary man of business would trust for the simplest speculation. If we have learned any one thing more certainly than any other, it is that we can only know about the proximate. We can take and take again, one little step forward. But all they do is to start from the ultimate. They know, forsooth, *what is,* and from that they deduce what must be."

"But you believe in God, Uncle?"

"I don't know what they mean by *God.* All theology is a vast fiction beginning from the wrong end, which prevents our finding out that proximate *higher* which we might have a chance of knowing. The fools," he muttered, "with the tools to their hands and with their eyes gazing to heaven—refusing to use them. Refusing till too late."

Thrilled to the heart with the fire of his gaze, Laura struggled with the enigma of his emotion. It was not helpless mournfulness that weighed him down—not despair; something else than the doom of the world filled his mind, something he did not want to tell her. All at once she said, guessing his secret thought, "Uncle, you could save the world!"

"What has your father told you?"

"Nothing. But I know you would not feel as you do if you could not."

"But no one will listen to me."

"Uncle, what is the good of hating anyone? They are all really in earnest. It is only because they don't understand you. Go and talk to the learned men who find out about the earth and the stars."

"You do not understand, Laura. The dogmatism of scientific men is stronger than the dogmatism of religion, because they can prove they are right. They can appeal to the evidence of their senses.

"I, too, was like them, and believed that I, my power of thinking, and every faculty I had, was produced by the things around me; that the processes I knew of would make me if only I sufficiently under-

stood them. I laughed, the idea of a spiritual existence apart from matter to scorn. And as to a third dimension, it seemed to me ridiculous to make an assumption for which we had no evidence. But I gained the acquaintance of a man of perfect loyalty and veracity, who was gifted with singular powers. He claimed to have communication with the spirit world, he showed me many things that people account miraculous. What I noticed was that these wonders were things which it would be easy for three-dimensional beings to do. Scientific men called him an impostor and cheat. But I knew him too well to join in the chorus. He made me believe in the third dimension by the only evidence I would accept—the evidence of my senses. And, Laura, I am proud to say I stood by my benefactor. It was the only act I can account worthy of a man in my whole life. It's a little thing you would say, just to declare that certain things happened to little bits of matter. But it cost me every friend I had. I could not even retain my poor and insignificant position. It is as much as anyone's professional reputation is worth to have anything to do with me. If I had an experiment as clear as day they would ascribe it to trickery. And, Laura, it is this truth that would save us all if men would only believe it."

"But," said Laura, "if you believe in spirits, the clergymen would listen to you."

"Much good that would do. I tell you, Laura, that those men are so versed in unrealities that, if anyone were to tell them of how what they talked about were possibly real, they would like to burn him alive, if they could still keep up their taste for that sort of thing. No, they have a closed system of their own; and their idea of thought is to try to make out exactly what is written in old books and find out whether this man or another really lived when he is reported to have lived. That's the kind of stuff they give to a perishing world."

"But if they make people better?"

"Yes, yes; it's their thought I'm thinking about; what they really understand. I've tried your father. I am always trying the scientific men, but they politely return my work. I know what they think of it."

"Uncle, I know someone who would help you."

"You are very beautiful," he said, as he drew her to him, "there is someone who loves you very much."

"Why do you say that?"

"My dear, I know it."

"Uncle, I will be frank with you; somehow I feel that I can tell you. I love someone very much. He has never said anything to me, but I

hope, indeed I am sure, that he loves me. I feel this way about it. There is a great fountain of love from which we all drink. Now I love many people and think about them frequently, but there is one whose being gives color to my every thought. Everything I think or do or say in some way relates to him; and when any beautiful thought comes to me, it turns to him. Now, Uncle, do not laugh at me. I am not investing him with supreme goodness, I simply feel as I have told you about him. And when I think of all these things and how much of his individuality has become the best part of me, I cannot help believing that he must in some way absorb something of me."

Farmer took her hand. "Thank you, my dear, for letting me see your heart. Yes, I think he must love you too. Do I know him?"

"He is Harold Wall," said Laura.

"I knew his father. After the great war he could hardly speak our language. He used a dialect which had sprung up in the long journey. To the end of his life, he was the same—rough, uncultivated, unable to adapt himself; men honored him for the way he had hurled his men through the deserts, but did not love him. His life was used up in that effort, as mine has been in mine. Perhaps his son will understand that I, too, have ventured and succeeded, when to everyone else I am a drivelling dotard, speaking in a scarce intelligible language. I suppose," he added, "your father does not look favorably on him?"

"He has never spoken, he is too proud. If he did care for me, he would never speak."

"That is well. Laura, something tells me that the tie that binds you two is very deep and true. You must not think, as other lovers often do, only of one another; but out of the great tenderness and truth of your affection, you must turn and give yourselves—give one another—to help and save us all. Now write to him, giving him a message from me. Tell him that he can save his country."

Laura's Letter

Lone Mountain

Dear Harold,

I am staying with my uncle. Before I left the city I saw a great deal more of my father than I used to. And I found that there was something weighing on his mind, something that he kept to himself. Once he was very angry because he thought I guessed it.

We have been living in ignorance of a terrible danger that hangs over us, over the whole world. My father knows of it, my uncle knows it. If you go to the Director of the State Observatory and tell him you come from my uncle he will not deceive you, though the danger is kept secret. It is this. The summers have been growing hotter and the winters colder, because we are attracted out of our orbit by Ardaea.

In a little while we shall be frozen to death or burned up alive. My uncle knows a way of saving us. Please come and let him tell you what it is. By himself he can do nothing, but with you to help him he can bring us to safety again.

Yours very sincerely,
LAURA CARTWRIGHT

If one of us had taken the journey Wall took in response to this letter, had passed the crowded cities, the straggling villages and solitary dwellings on our way, we should have felt a strange sense of isolation—as if our unshared knowledge was phantasmal, and all those unconscious people in the possession of the truth, not ourselves.

How could the age-long routine of business, barter, trade—the intense and urgent solicitude of every man in his own affairs—how could the absorption of each man in his own microscopic corner be interrupted! By dint of their intense preoccupation in their own individual affairs, the race has surely won the right to have the basis on which all rests undisturbed. The firmament, the arch of sky, the mutations of the seasons, the fabric of the earth, they at least must be secure. But Wall had wrung the secret of State from the unwilling astronomer and accepted it. He believed in the interplanetary vicissitude and was prepared to consider means of averting it. There was no lack of events which he could have looked on as ominous and significant. Great storms had beaten on the shore. A tidal wave of unprecedented magnitude had caused serious damage to the ships prepared for his expedition. The frightened colonists refused to sail till the sea was settled to its usual calm. But his attitude, his readiness to believe and act upon a theory can only be explained by the history of his people. We must remember that Unaea owed her existence to an idea—it was the idea of the circularity of the earth that saved Unaea from destruction at the hands of the Scythians—and therefore the Unaeans had a different attitude with regard to ideas to that which we have, for we belong to the barbarian hordes who swept off the face of the earth the people who had ideas—the Greeks and Romans. Ideas are to us of incidental assistance, but we feel that essentially we can do very well without them. The Unaeans were different, they had a

faculty of realizing and acting according to ideas which seems strange to us. Our history is as the Astrian history would have been had the Scythians overwhelmed Unaea, and plunged the nascent star of civilization into long centuries of eclipse.

At this epoch of his life, Wall was free from those charges of unbridled and self-seeking ambition which were afterwards levelled at him. His life, if obscure, had been simple and straightforward. For so young a man, he exercised a remarkable influence over his comrades, due perhaps to his power, which showed itself so often at a crisis of coming to an unexpected but irresistibly incisive decision, sweeping the minds of all along with his own—an influence perhaps due, in some measure, to the hidden passion which lay behind all his frank comradeship, giving a touch in his intimacies of that zest for the unattainable, that reaching beyond the obvious bounds of fate which lies latent in everyone.

Some would look on him on this journey as filled with an unscrupulous ambition, preparing to strike with the subtlest instinct of success. And this no doubt is true. Strictly speaking, his course is indefensible. But there is another side. Let us look on him that last night of his journey as he hurried on.

Slowly Ardaea rose, the hymned of mortals, the divine orb, the legended cold lover of the earth, towards whom poets had ever turned lavishing their adoration. She slowly rose, strangely ardent, and burning bright, for at last that cold heart was touched: the chaste and solitary, the huntress of the skies, had turned from her lonely path and, responding, was already swerving in one moment of the giddy whirl of passion to draw her earth lover to his endless death. But innocent! Away with the fables that lend the appearance of purpose to the course of things. In the appointed revolutions of the orbs of heaven, in those great secular changes, there is but inevitable law, and in the cold rhythm of the cosmos, the warm pulse of heart, the plan of mind, and all the fabled legends of the soul of things, is but as the plash of a pebble in the ocean, signifying nothing.

Yet wherefore this throb? This life passion that he felt rising within him as he drew near his journey's end? In the suffusion wherein all his being lost itself in another's, in this was there not something as great as in all the world's inevitable course?

Thinking of her and all she meant to him, he entered on a different path, a different way to that wide contemplation of vastness, but in this intimate, most secret, and real communion he arrived at some-

thing as true and as strong as all the substantial distant phantasmagoria of earth and skies.

Did she not tell him there was hope? Himself and her in the face of this great catastrophe, were not alone. Love and trust and hope had ever been, had ever faced the vast mechanic universe. What of the age-long revolutions of the planets, had there not been age-long efforts of true men. Incalculable vast forces went to the swinging of the orbs of heaven, but also incalculable vast forces, age after age, and generation after generation, in the beating hearts of men, labored there also, built up their edifices, prepared their powers. And in the army which had saved Unaea, which, now abandoned of its use still strangely lingered on, for all its forlornness yet capable of predominance absolute, perchance that army now by gathering all the forces of the land, gathering them from ineffective hands, might save the earth—what else than some such message could her letter mean?

Harold found himself in the presence of an old man, bowed and emaciated, but of a ponderous brow and keen gaze, and his love stood by silent. The old man asked him: "You have questioned the astronomers?"

"Yes, they have left no doubt in my mind."

"In the greatness of our peril," said Farmer, "all that human ingenuity has devised stands for nothing. No known force can alter the orbit of our earth. The event is hopeless as far as our science goes."

"That is the opinion of those who have studied the question."

"But it is hard to set a limit to what is possible. What men think possible depends on two elements, not one. It depends on the facts and on their ideas. Now, science has been concerned in developing a certain limited range of ideas—they are a few out of the many ideas of the past—just the few which we justify by observation and experiment. But there are many more ideas than these, and I believe that our path lies more in acquiring new ideas than in the one we have trod for the past few centuries—in working out the consequences of the ideas we have.

"There is one idea which I have been trying to live into all my life, and which gives a perfectly new range of thought and physical possibilities. It is the idea of a third dimension. According to it, when you think you are in empty space you are really not so. To prevent your moving in the third dimension there must be some physical cause, a source of resistance. This is the alongside being, a substance with which you are in contact whenever you move, which you never can be-

come aware of because you never leave it, you are like a particle slipping along a smooth edge—the edge prevents its moving except in a line.

"Now, along this alongside being, there are all the directions of motion possible which we can point to, and by acting on this alongside substance we can hinder and change our movements. The means, and the only one by which we can escape the catastrophe, is by acting on this alongside substance to deflect the course of our world. This can be done. I will explain how."

Harold answered, "To say that there is something besides space which stretches infinitely all round us sounds to me absurd. You have a scientific theory, put it before the learned bodies, you can convince them if it is true."

Farmer made a gesture of resignation, but Laura seized his hand in her warm grasp and smiled encouragement, whispering, "He only wants to find out what you have done already."

The Orbian

Wall found it an easier task than he had expected to gain Farmer a hearing, there was a vague sense of uneasiness and expectation spread abroad. Some vast excavations undertaken by the government in regions where no one had hitherto thought of looking for coal or ore had given rise to a sense of mystery and secrecy.

He himself said nothing of the threatened calamity, but chose auditors of whose judgement, discretion and silence he was well assured, to hear Farmer's message.

Most unexpected of all his successes was the ease with which he gained a hearing for Farmer from the head of the Orbian church. The incidents of the scene are worth recording, as they show the difficulties which attended the promulgation of Farmer's theory, and the arguments by which he endeavored to overcome them.

The Supreme Pontiff of Unaea, the head of the oldest religion in the State, in which the impulse towards efficient organization and the detachment from the ordinary life of men on the part of the clergy had gone hand in hand through century after century, sat in his audience hall.

Around him were priests who had made each branch of human effort or learning their life-long task, each one tempered to the maxim of absolute obedience, so that when the fiat had gone forth from the frail

man, the vice-gerent of God, no doubt or hesitation crossed their minds, but that became accepted fact which before they discussed with the utmost freedom and subtlety.

Before him stood Farmer, seeing in him the embodiment of all he hated, the supremacy of something else than reason, the haughty claim to judge otherwise than by the proximate—the prime cause of all error, the reason why the misdirected efforts of men hung bitter and bereft, cajoled and chidden from the true path to the apprehension of the world.

Standing proudly before the frail man—who, sunk in the great throne and robes of state, received him with preternatural silence and abstraction—Farmer began to tell of the astronomical situation and the approaching destruction of the earth.

The pontiff's eye flashed for the first time. "You may assume that as known," he said. "The time for the harvest is brief, and the laborers few. I pray you use all brevity."

This calm acceptance of the situation at the center, whence not so much as a ruffle had spread over the surface the church showed the world, impressed Farmer. They knew all and showed no signs of trouble! How different to his own agitation, the attitude of these men, who but saw in the end of all a more pressing call to their work!

"Holy father!" he said, using the mode of address of the faith, "I will be as brief as is consistent with the difference of our ways of thought. I come to you because you control the effort of half the world, and if you consent to direct it in a certain way, you can avert this calamity."

"Speak on, my son."

"The beginning of my thought was that everything is in space.

"Every person with whom we come into contact we know as a being in space; every act of our practical lives, every thought we have is derived from things and persons in space; even the revelation in which you believe has the same origin, it comes from a man who was seen and touched.

"And if everything real that we know is in space then that which is not in space is not real. An immaterial existence is nothing. Hence it seemed to me that the only way to know more—to really know it, not imagine it—was to acquire more and more knowledge of things in space.

"I would admit that was much in the universe, beings, persons such as you claim in the traditions of the church; but the only way to know them is to know them as space beings.

"And with this view, the supposition that space had more than two dimensions seemed to me important. Possibly your miraculous accounts might be distorted, and fantastic views of real things and beings we might rationally know in this higher space.

"Possibly, also, much that is unexplained in science might be found to be obscure because of this same fact of space being three dimensional.

"But I was in this position.

"Imagine a man in a society where justice reigns, if he has no sense of justice how will he recognize the part it plays in the institutions of society? It would be no use his saying that everything that is inexplicable is caused by justice. For there is, no doubt, much that he does not understand of every kind.

"The only way for him to learn about society is for him to form the sense of justice in himself, and when it is formed in him he can recognize the workings of justice about him.

"And, in my case, I had no sense of the shapes and movements that could exist in three-dimensional space. The only way possible to tell whether there really were three dimensions was to form the sense of three dimensions in myself.

"Accordingly, though only a two-man myself, I made an account of the simplest things a three-man would have before him. I made objects which represented what a two-man would know by sight and touch of a three-man's objects.

"And I found a sense of three-dimensional forms and motions wake in me. They came to seem quite natural to me. It was as if I were really a three-man, and only confined by the condition of my bodily experience to a two-man's thoughts.

"Starting from this assumption, giving the necessary labor to develop my sense of three dimensions, I have come to recognize clearly that I am really a higher being in face of a higher reality, and can discern something of the undreamed of range of power and opportunity that lies before us.

"What is the relation of the three-man to our bodily frame? We know that we do not perceive outward things directly. Tracing what happens when we see or gain any knowledge through our senses we find that certain changes take place in us. It is these changes that affect our consciousness, not outward objects directly. The incidence of the world of things in that which properly perceives takes place in processes of extreme minuteness."

One of the professors of the sacred college interrupted Farmer at this point by saying:

"You come then upon the mystery of thought. Thought, personality, the self is immaterial and cannot be explained by any of the principles of physics."

"No," replied Farmer, "all I do is to say that before you come to the mystery of self and personality, there is an intermediate domain to explore. Without touching on the mystery of thought you can examine the processes in this domain.

"When you come to the minute operations of nature you come to actions in threefold space, and that which really animates and directs our corporeal frame has this very same kind of activity.

"The three-man is small compared with our bodies, but mere size is no bar to any complexity of structure. We are three-men directing the activity of corporeal frames limited to twofold movements.

"But then the question comes: If there are three dimensions to space why do we only perceive a two-dimensional world?

"There can only be one answer. Because we are limited. In these bodies our freedom of motion is hindered, we can only move in our corporeal frames in two dimensions because something prevents us, prevents all things, these planets, worlds and suns from moving freely.

"That which prevents us I call the alongside being. In whatever direction we point and look we do but choose a direction along its boundary, never a direction into it or away from it. And, with the recognition of this alongside being, at once a new field of possibilities opens.

"If we were free in space there would be no way to influence the course of our planet.

"But, being always in contact with this alongside being, if we were able to drive a spike into it we should retard our motion. Also, if you study three dimensions, you would intuitively understand that there is the possibility of pressing an edge into it, so that by properly placing the edge against the alongside being the motion of one's body could be deflected."

At this point Farmer paused, expecting an objection, for this possibility which we can express so easily by the word "skating" was one which offered great difficulties to the Unaean comprehension. We see that a body sliding on a smooth surface can easily be deflected. For instance an ice boat, on the surface of a frozen lake, can have its course altered by altering the inclination of the edge of a blade which

bears on the ice. But to the Unaeans such a process was entirely inconceivable.

"It can be admitted," said the professor of the sacred college, "that in a space of the kind you imagine there would be possibilities of varied kinds, and the one you suggest may be one of them."

Farmer continued: "I can assure you that this possibility exists, and I connect it with the persistent accounts which have been handed down of people's rising in the air, of a power of rising above the influence of the earth's attraction.

"The origin of all these accounts is, I believe, an obscure sense of the existence of an alongside being and a hidden feeling of a possibility of directing ourselves otherwise than by contact with anything we can see with our eyes or touch with our hands.

"Such a power of directing the movements of our bodies up or down is trivial and unimportant. But it has a bearing of vast importance and consequence. We are on the earth, our bodies are parts of its mass, and any direction we could give to the motion of our bodies, by uniting together the efforts of all men, we could impart to the earth.

"It is plain that if we could direct the course of our planet we could avert the dangers which our too-great proximity with Ardaea will bring.

"We have no outward means of acting on this alongside being. We two-men have no such power. But the three-men, who are our real selves, these three-men have the power.

"By thinking of rising, of soaring through the air, the three-man, who is my real self, calls into play activities of his own.

"He acts on the alongside being, he puts an edge into it, so that the motion my body has in common with the whole earth is deflected and I have an upward tendency. I have put this to the proof. I have found that my weight becomes less when I think such thoughts as I have described.

"Now, if all men were to join together in worship, thinking of themselves as rising, as soaring like angels through the air, they would produce a force which would be enough to cause a certain deviation of our planet's course, a very minute one it is true, but all that is needed is a very minute one. Thus we could pass Ardaea in safety."

Farmer's theory was two-fold, first, that by a grouping and rearranging of the molecular structure of the brain, such material changes could be effected as would cause a body slipping over the surface on which all Astrian things moved to be deflected in its course; and,

secondly, that by thinking certain thoughts such molecular changes were produced in the brain matter of the thinkers. He postulated an accord between the conscious thought of rising and soaring, and those minute changes which by a process quite unknown to the thinker would bring about the realization of this thought of rising and soaring.

To convey this idea was a task which presented almost insuperable difficulties. He had exhausted the known resources of the Unaean language, and had said all that could be said to men who had not followed his path of thought. Therefore, addressing the supreme pontiff, he concluded with the following words:

"I come to you, for you have the direction of the religious effort of half the human race. If you would decree a form of worship in which certain thoughts should be followed at certain times, and all the fervour of your congregations united, you would set those processes at work by which the three-men, our essential selves could save the world from its approaching catastrophe. . . ."

"Holy father, have you any question to ask me?"

"No, my son, receive the blessing of an old man who, like yourself, has striven to be faithful to the task allotted him."

And so the conference ended.

Laura found her father agitated beyond description at her absence.

"My daughter," he said, "I have long been intending to speak to you on a very important subject, but have put it off. I must not let this occasion pass."

Laura told him she knew everything.

"It makes my task the shorter," he replied, "I have decided that the time has come for you to take a husband."

"Not now!" she exclaimed.

"Yes, my girl, some few of us may expect to survive. I have prepared subterranean chambers, which will be stored with all necessary provision; there some of you can pass the time of transition, and emerge to the new order of things.

"Papa, I'd rather die than be shut up like that."

"It is not what you wish. We must strive that some of the best of us, those most fitted to carry on the destiny of our race, shall survive. It is not your part to question—the decision lies altogether beyond your power to alter. And I can easily remove any hesitation you feel. I know you were attracted to that man Harold Wall. Whatever you may have felt for him cannot remain alive a moment longer when I tell you he is making use of this approaching calamity to stir up sedition. He is

consumed by a reckless, unscrupulous ambition. He has taken hold of the fantastic moonings of your poor uncle and, using them as a lever, has tried to persuade a number of weak-minded sentimentalists that there is some way of avoiding the danger. He is trying to make this world-peril the occasion of promoting disorder and securing his own ambitions. He has been unceasingly active in drawing closer his relations with his fellow-officers, trying to make them traitors to their oath."

"Papa!"

"Yes. So well has he chosen his confidants that we have had no direct evidence, but the army is penetrated with the knowledge of what I tried to keep secret. The panic that reigns has enabled us to pass special measures. Before nightfall he will be arrested, if he resists, he will be killed on the spot, no mercy will be shown him—he will be sent to Septentraea—not as leader as he might have been—but a convict. You can understand the wickedness of the man when I tell you that we have decided to employ a company of Scythian soldiers. We cannot tell to what extent his machinations have gone with the regular troops."

Her father turned on her the full force of his implacable regard. She felt the iron resolution and the unyielding purpose by which he had won his way and beaten down every opponent. In her powerlessness her only thought was of some means of warning Harold.

He took her silence for submission. "One who has long loved you and who, I believe, has not been without encouragement, is here today to urge his suit. You will listen to the dictates of your own heart and of my wish if you accept him."

But, Papa, I have given encouragement, as you call it, to so many."

"You must know who I mean—Mr. Forest."

"I like Mr. Forest very much."

"Let's have no beating about the bush, my girl. You agree to accept Mr. Forest?"

"How can I say before he asks me."

"No nonsense, girl."

Laura's heart quailed, she was desperate, she must send word to Harold at once. What if she bound herself for life if only she could save Harold now!

"Yes, father," she said.

"Understand me clearly, Laura, you were seen talking to this Wall only today. I will not have you implicated. You will not be allowed

out of my sight, or away from someone equally competent to guard you till you leave my house for good."

"You don't place much confidence in me, Father."

"How can I with a telltale face like yours—no, Laura, you will thank me some day." And he left her.

When she looked up, Edward Forest was before her. "Since you have consented to see me I begin to hope. You have known that my love is yours, will you accept my lifelong devotion?"

Her silence gave him courage. He kissed her pallid unresponsive lips. It was too much—what was all her assent, her submission for if she could not save Harold. Just because her father had taken prudent precautions to prevent her betraying him she felt outraged.

She pushed him away and said, "I wish I were dead."

"Laura, how have I offended you? Your father told me that you did not dislike me."

For a moment she struggled—he looked so forlorn, and a word from her would bring such joy into his face, he would do anything for her—she was sure she could cajole him into taking her message for her. But into her mind came Farmer's words, "Truth and straightforwardness find a way when all other means fail."

She laughed merrily—"Don't look so dejected, Edward," she said. "Papa tricked me, I said I would marry you just to get a chance of speaking to you alone. I suppose if you insist on it you can have me, but there's something much more important than that."

"Nothing to me," he said, "if the world came to an end tomorrow."

"But it isn't coming to an end, we are all going to live happily ever after and you can help, help more than any one man living—you don't know the good tidings and the hope, do you?"

"No, Laura, I've heard that there can't be any."

"Then, Edward, you believe that God made this beautiful world just to be destroyed like that!"

"It makes me not believe in God."

"Edward, I will tell you all about it. You know that in old times men had messages from God that told what His will was."

"Yes, I've heard so."

"And did you ever wonder why it was always through men, not by some great being appearing as big as the sky?"

"No, I have always taken those things on faith."

"Well, I will tell you why. What do you think you are really, your

very soul? Don't you think it is something like your body, only filmy and shadowy, not exactly real, but shaped just like your body?"

"Yes, I suppose that is how I think of it, if I think of it at all."

"But that's all wrong. I'll tell you what my uncle says. He has found out that what we call all of space is only a little bit of it. And we are curiously confined in all the movements our bodies make. There are really three dimensions not only two. The real world is a world of higher space. If we want to think about ourselves in a world of higher space we must go the other way first, and think about a being in a world of lower space. Think of a little creature that is confined to living in a stright line. Such a being would not think of its support at all, but would think what was in front of it and behind it made up all space, and would not recognize that it was on something. So we, in the threefold world, are supported in a direction we do not know. And just as the line being must really have some thickness, so we have a thickness in a way we cannot point to.

"Now you know that we are told that our souls have come into a world of matter and have taken on themselves the limitations of it. The case really is this: Our souls, which are these higher beings, have come into a part of the universe where the work to be done is in this twofold space of ours. It is as if one of us went into a very narrow tunnel where there was only one direction.

"What this work is we do not know yet, but the beginning of it is to conquer the difficulties of the world, and to live all together unitedly, so that when we know we can act together. And the souls that come into this world form united bands, a great many of all degrees join together and animate a body, but all of them are under one soul which is one's real self. And this real self soul directs all the others in the body, like a captain directs a lot of men in a ship, each has its work. And the directing soul that has the business of directing our actions, this soul almost forgets its true being; it is very faithful to its work, and is absorbed in it. It thinks it has only two dimensions, and nothing it can see reminds it of its true existence. We are hidden away from all the other souls of the universe, like a man in a narrow tunnel would be from us. If the other souls want to speak to us they have to enter into our conditions, they have to put on one of our limited bodies—that is why the voice of God has always come through men. And now God knows that there is a great danger which will spoil all the work He has sent us here for, so He has sent a soul with a message of what our real condition is, so that recognizing our own

true way of acting, not the body's but the soul's, we can have quite new ideas of working, and save ourselves."

"But, Laura," said Edward Forest, "what a curious idea you have about the soul; the business of the soul is to do right, to grow good and improve itself."

"No, that is a very poor kind of soul that thinks that," said Laura, "all the men with big souls try to do something in the world—like my father does. He has a big soul but a very mistaken one; of course, good souls would not demean themselves by doing wrong things, if they can't get what they want honorably, they would rather give up and let another try. There are plenty more. But, Edward, say you believe me, your soul must feel that what I say is true."

"I don't see that it has anything to do with the collision with Ardaea," said he.

"Ah, that is exactly what it has, for we have never thought of that support we are on, we are against something and there is a way of holding on to it so that we can alter the way the earth is going."

"Are you sure of this, Laura?"

"Yes, I am so sure, Edward, and I want to do something to help it on."

"How do you mean?"

"Oh, Edward don't you understand, my father is going to stop it all, and I trust to you."

On Forest's face came an expression so enigmatic that Laura stopped—"What is the matter, Edward, are you angry?"

"No," he said, "it was a conflict of feelings that made me frown. I do not quite know where I am."

"Here, and ready to help me," she said.

"Yes, of course, Laura," he said, "but there's something else."

"Do you not think," he went on, "that your talents would be thrown away in the domestic circle? There is the making of a most eloquent professor in you."

"Oh, don't say that," she replied.

"When I come home I shall have to call up all my knowledge of mathematics, ghosts, astronomy, and theology too, Laura. I think it would be very exhausting."

She looked at him in alarm.

"I made a proposition to you just now," he continued, "but I did not know your talents. I wonder if you could be induced to allow me to withdraw my proposal."

She looked at him puzzled, a strange mixture of relief and consternation came over her. To have no power over him at all! To have shrivelled up all his long devotion just by a few words! She couldn't bear it—and yet, not to be bound at all!

"I am waiting for your answer, Laura, may I withdraw?"

"Upon conditions," she replied.

"Yes," he said, "on conditions—I will tell you what they are—that I shall be your very best friend, and that I shall help you in every way I can. What is it you want me to do now, at once?"

"Oh, Edward," she said, with tears of gratitude flowing, "I shall never forget how you understand me."

"Laura," he said, "you are wonderful, you remind me of the prophetesses and sybils of old times. You have a great career before you."

"Now, tell me," he went on in a business-like tone, "I judge there is something very pressing—something you want me to do at once."

"Yes, Edward," she said, "it all depends on you. My uncle knows how to save the world, but gave it up as hopeless to make others join with him till Harold Wall came to him, and told him what to do. And Harold has gained him a hearing. Now my father does not believe in any of it. He thinks that Harold is taking advantage of people's terror to prepare a revolution. He has an order for his arrest, and is going to send a company of Scythians to arrest him. Now you must warn Harold."

Edward Forest frowned. "With all his judgment, your father is in fatal error. I told him this enrolling of those barbarians as a special regiment was a mistake. There is no doubt of the loyalty of the army, and if there were, such a sign of mistrust would be most foolish. I'll go and tell Wall what you say, but your father is not the man to threaten before he strikes, the blow has probably already fallen."

"No, Edward," she said, "Not yet; I know not yet. For he is keeping me a prisoner here."

I'll go at once," he replied, "don't be too much alarmed."

In the Orbian palace, Farmer stood before the mighty assembly of prelates, priests, and clergy of every denomination. The influence and force of their natures so alien to his own seemed to make no impression on him. In a few simple words he placed his thought before them in the manner he judged they would best understand it.

"In the history of our race who can say what is the order and what is the plan by which we came upon these great facts, the knowledge of

which reveals to us what we are and gives us mastery and control of our fate?

"There is no order or plan that I can discern save that somehow, when the time has come, we have seized on that which it is important for us to know.

"Thus, in the early ages when our mechanical powers were nothing and when our rational knowledge of nature was ridiculously meager, you—those with whom you are united as one in the succession of your efforts—you, I say, discovered the soul. There is something in us superior to the body that stands apart, directing it to higher ends than its mere self-preservation. But this discovery, great and all important as it was, was not complete and rounded off; it depended and depends on an inner intuition of man's nature, it was not and is not connected with the rational system of things as known.

"And gradually as the examination into the nature of the material world around us progressed, this discovery of the soul, this near and intimate knowledge became incongruous with the record of our senses.

"For we found no real place to which the soul could go, as in the earliest ages was implicitly believed. We found nothing in the body but an animal organism. And hence to this day when you give your message to the world you rely on other evidence, on other principles than those which govern the reasonable conduct of affairs.

"But I have discovered the soul again. I have discovered it not by the way of inner conviction, not by the overmastering energy of its verdict on our conscience and on our actions. I have discovered it as a real being giving as much for the outlook on this physical world as it has done to enlarge our prospect as human beings. Just as the intuitive knowledge of the soul has raised our moral being above the ways of animals, so does the rational knowledge of the soul raise our intellectual being above the ways of things.

"For in our thought we have lived a life of acquiescence in the subjection of the body, whereas, when we know our true being we find that we—our essential selves, our souls—stand clear of it. And this knowledge comes to us now when it is necessary now when even to preserve ourselves we must rise above the conditions of our subjection.

"I would lead you to the greater higher world. And do not turn from me when you think I speak of insignificant things. You tell us of highest love revealed in common offices. And we reach the freedom of our intellectual being through thoughts of common and insignificant things.

"I tell you that the thought of old time of the soul as real is right, undivested of any of the garb of our life, nay rather clothed upon more and more with an indescribable fullness of being—such you should know it—follow me and you will know the soul rationally and deliberately, as now you know it outleaping the slow steps of reason.

"The path is this. Think of a being confined to a line. You will perhaps think of an insect that cannot rise above its support, but the illustration is not correct, for inasmuch as the insect feels the support it is on, it has a knowledge of two dimensions. A being confined to a line would have no idea of anything except that which was in front of him or behind him in that line. And by the very condition of the limitation of his being he regards certain operations as impossible.

"The line being has two extremities which we may call the head end and the tail end. The head points one way and the tail another. By no possibility can the line being interchange these directions. Given two line beings, the head of one pointing in one direction, the head of the other pointing in the other direction, to them it would appear to be impossible that they should be so placed that their heads could point in the same direction.

"We see that they could easily be placed pointing in the same direction, we can turn them round so as to point the same way. We can do this because we can use two dimensions. Not being able to move in two dimensions they think this is an impossibility, they think it is of the nature of space that there should be this impossibility. But we see that the impossibility to them, of disposing themselves so as to point the same way simply shows that they are limited, that they do not possess in their bodily movements the possibilities that actually exist.

"Now coming to ourselves we find an impossibility. Think of equal right-angled triangles symmetrically placed about a straight line. We may shift these triangles for ever but cannot make one occupy the space of the other—there is always some incongruity.

"Now I say that this impossibility is not a real impossibility—it is a consequence of our limitation. If we could move in the third dimension we could easily place one of these triangles so that it would fit in the space of the other. Just as the impossibility which we find is a sign of our limitation.

"And now consider the line being. The recognition of the second dimension would make him aware that he was always in contact with something—his world is not a world of empty space but of support upon something—there is, where he would think it was free space, an alongside being.

"And similarly with us when we stand upright and move our hands we think we are in free space except for the earth rim on which we stand. But it is not so, there is an alongside being for us too, and however we move we are in contact with it, along it we move our arms whatever way we point.

"And existence itself stretches illimitable, profound, on both sides of that alongside being. Realize this, it follows so that no one can doubt it from what I have said; even begin to realize it and never again will you gaze into the blue arch of the sky without an added sense of mystery. However far in those never-ending depths you cast your vision, it does but glide alongside an existence stretching profound in a direction you know not of.

"And knowing this, something of the old sense of the wonder of the heavens comes to us, for no longer do constellations fill all space with an endless repetition of sameness, but there is the possibility of a sudden and wonderful apprehension of beings such as those of old time dreamed of could we but look athwart this all of sense, know that which lies each side of all the visible.

"Such an apprehension lies in the future—what is the meaning for us now?

"To interpret the mystery of our being, to discover our relation to the wider universe, go back and ask yourselves how a line being could exist. No real being could exist in a line. A real thing or beings must have all the dimensions there are. But a real being like ourselves, possessed, as cannot be denied, of two dimensions, can be put in such circumstances that it has only a one-dimensional experience. It can be part of a structure or organization which is limited to one-dimensional movement.

"Think for instance of a ship which moves on the water. It can only move in a line. Imagine its captain directing it, if he were unconscious of his own movements and simply had regard in all his thoughts to the motion of the ship and identified himself with it, he would look on himself as a line being. But if by any means the idea came to him that there were two dimensions, if he could interrogate his own bodily consciousness, realize himself separate and apart from that which he directs, he would have plenty of experience of two-dimensional movements. All he would have to do would be to wake himself up to his own essential mode of being.

"And similarly with ourselves.

"We, essentially, are higher beings possessed of a higher kind of action than we realize in our bodily movements. This being, that is

essentially ourselves, is the soul, and just as it has waked up to knowledge of itself in conduct, has recognized that it is the master of the body and stands apart from and superior to a mere animal life, so it now is ready to wake up and recognize that it is superior to the ways of things. The movements of the body are inferior to, less ample than, our own movements. The mechanics and movements cognate to the soul are superior to those it can apprehend through sense, are superior to those it sees through the bodily vision.

"The proof of this lies in trying it. I have waked up my soul, and I can think of three-dimensional things and how they act and react on one another.

"And I have found what lies beyond all I have been telling you, that just as the captain of a ship has an activity independent of the ship, so our souls have an activity independent of the body. Our souls can act on the alongside being. We, the earth, and all slip rapidly over the alongside being in the course of our planet's motion. In any movement we make with our bodies we do but act on things all equally subject to this motion. But our souls can act on the alongside being directly. And by this action we have the possibility of influencing the directions of our movements apart from pushing against or pulling anything we can see.

"At present exactly how this is done is obscure, these organs in the body by which the soul effects this result are too minute for us to distinguish them, all that we know is that we can rationally predict their existence. And those old legends of men raising themselves or flying through the air have their basis in the very fact of a relation to the alongside being which enables a man by the activity of his soul, not directed in the way of any of our ordinary bodily manifestations, to change the direction of his motion relatively to that of the earth.

"If, filling my mind with devotion, I think of myself as soaring, as rising like an angel through the air, my soul does that which would make me rise, altering my direction by acting on the alongside being.

"If all men were to have the same thoughts, then all of them would tend to rise, and the united force would be very great, enough to influence the course of the earth in its orbit. The force would be great enough, but, unless regulated, that which was exerted at one time would neutralize that which was exerted at another time.

"If, however, we properly choose our times, then by devotional exercises, by the whole human race uniting together in the thought of a glorified rising and soaring above the earth, we can influence the

course of our planet, we can effect that small deviation of our course which will enable us to pass Ardaea in safety.

"You are now face to face with the question. The danger is real. It forces us to leave off trying to explain the world by our ideas and try instead to conceive the reality."

Now there were in Astria a class of philosophers who looked on the all as one great being bent on his self development. They said that different individuals were but his ways of imperfectly and partially apprehending his own thoughts. Among some denominations these philosophers were considered to have made a valuable contribution to the support of religion, and were regarded as very profound. One of them rose up and said: "We can never go outside our own ideas; it is absurd to speak of reality as if it were any different from an idea."

"I will not stay," said Farmer, "to discuss this question, which is merely one of words. I have found for my part, that words never have any definite meaning when you take them close, but held at a distance they serve very well to point out a general tendency, or suggest a contrast. And the contrast I alluded to is plain. From our two-dimensional solids we have, we obtain by abstraction, the idea of an edge bounding them. And from this edge or line, we can make the further abstraction of a point. We can try to explain the world by using these abstractions, these ideas, or we can, on the other hand, try to conceive that with regard to which things, as we think them, are mere abstractions. We must form ideas which we have not got. And in the face of that necessity, which has occurred before now, I believe that the rough and ready way we have of contrasting our ideas with reality points to and indicates a distinction in our way of proceeding. To cope with Ardaea we must obtain new ideas, for with our present ideas, as you know well, there is no possibility of avoiding destruction.

"I have found that by thinking certain thoughts, I can volitionally direct the activity of that real being, my soul. I can alter my weight. There is a power in everyone of doing this. Now an alteration of weight can only come by our acting on the alongside being, and this is the very action which is necessary to alter the course of our planet. Our souls have this power. By introducing the thoughts which produce this action into your forms of worship and inducing your congregations to follow them with fervent piety, you can alter the course of the earth and pass Ardaea in safety. Every being must sometimes come to reckon with the absolute facts of its being or perish. That is our case now. Though placed as far as our bodily vicissitudes in a two-

dimensional state, we must act according to the three-dimensional reality.

"And as your congregations by this worship gradually bring safety to the world, you can inculcate the truth, which they will at first act on unreasoning and blindly; you can tell them of a real soul. You yourselves will not grope blindly about in tradition, but discover more about the soul, you will approach its study, not from the side of consciousness only, but as an objective reality. And if you hesitate to take my view, because you think you demean the soul by considering it small, you must remember, that though small in any way you can measure, yet it has a thickness in a direction you cannot point to. There is more in a square, however small, than in an infinite line. And so one soul has matter enough in it to make up endless universes such as we conceive them."

All through this speech the Orbian pontiff sat unmoved; his face, pallid and emaciated, was as that of death, to which no words can come, and his gaze was as if sunk in some profound region from which it would never return.

The first to take up the word was a bishop of the Literal Church, whose doctrine was a faithful adherence to the sacred texts.

"Brethren," he said, "in commencing our deliberations on what we have heard, our first thought must be, 'How does it agree with the message we are pledged to deliver to the world?' Ours is the message of salvation, not of mere earthly good, and we must not let any promise, however fair, quench the light we have, or tend to weaken us in the promulgation of the truth."

Farmer broke in: "Yes, you see that it destroys every vestige of what you teach and inculcate. For want of anything else real to think about, you identify yourselves with these bodies, and all your notions of right and wrong are centered in the bodily relations. You believe the earth is a place for the display of virtues. What you eat, drink, how you marry, get unmarried, how you tend your own and your neighbor's body—that is all your thought. Not a glimmer in all you teach of the real work of man, but pitiless immersion in the utterly insignificant."

Wall could detect no sign passing from the Orbian chair, but some indication must have been conveyed, for a great prelate close to the pontiff rose.

"Our thanks are due to Mr. Farmer," he said, "for putting so lu-

cidly before us the real question for our consideration. Mr. Farmer has arrived at a new conception of the body and, if he is correct, it will require a reconsideration of our ideas with regard to the soul. These two aspects can never be confounded, and his criticism of us in the light of his new conception of the bodily, will be most welcomely received. But that question has nothing to do with our present deliberations. They are, I take it, directed to our practical action."

These few words sufficed to turn the discussion to relevant topics. A period of earnest debate set in, and the minds of the participants were so absorbed that they paid but scant attention to the shaking of the walls when the earthquake came, and paid no attention to the messages which told them of the disturbed state of the populace. One speaker after another gave his reasons for and against Farmer's views, and gradually the sense of the meeting came to declare itself in favor of making a public exposition of them in every part of the earth, and bringing all possible pressure to bear on the government to appoint a commission of enquiry. A motion to adjourn was proposed. But Wall stepped forward, his heavy sword hitting the steps to where he descended in their midst.

"There is a voice," he said, "which speaks within every man, telling him at the last extremity what to do. That is the voice of God. And in you is the voice of God to men. Many times before now you have declared it. It is no time to turn this way and that. Either this man is of God and we believe him, or he is of the devil and we leave him. Before you I lay down a staff"—and he made a gesture of flinging something before them—"it is an iron staff and whom it smites it crushes. It is the army. Its honor is in obedience; if you command it in the name of God, in whose name stands the vow of every commission, all other allegiance is dissolved. As one man waiting to hear the voice of God, the army stands before you."

The solemnity of the moment grew clear to them, as the indescribable weight of the man who had spoken pervaded the silence. Almost it seemed as if no breath were drawn in that vast gathering. One word would unchain the deep. From the loosening of that incalculable power embodied in that one man, the men of peace shrank, appalled at his simple words. . . . The Orbian pontiff and all his priests, prelates, and monks had arisen. From the lips of that frail old man, came a voice clear and solemn.

"God has spoken," he said, "through the voice of the servant whom

he has chosen, he has saved the world." Then, placing his hand on Wall, he said, "I absolve you from your oath. Take the message of Salvation forth to the world."

[Wall takes control of the country and the Unaeans begin a regimen of daily prayer according to a schedule designed by Hugh Farmer. The prayer consists of thinking one is an angel soaring upward towards God. Meanwhile Cartwright seizes the arsenal and tries to start a counterrevolution, but Wall overpowers him. For awhile Laura and Harold each think the other doesn't love him, but things get straightened out. Finally it is time to see if Farmer's plan has worked.]

Conclusion

Observations of the utmost precision taken daily for the space of two years, were submitted to the closest scrutiny and worked at with the utmost care by skillful calculators. And as a result it came out that the planet's course had diverged by a sensible fraction of a degree. Shown, made manifest, marked clearly as by a finger pointing to it, revealed by their changed direction in space, there came to all a consciousness of their collective will. It was as the awakening of a soul to its embodiment in a corporeal frame, the thrilling joy, the wonderful, the inexplicable power of movement—this had come to Unaea. The inhabitants of Astria knew that in their cooperation together, a being transcending them had begun its autonomous course.

The ranks of sound understanding crumbled, the platform of educated opinion was swept away.

"*Et pur se muove,*" the great words which mark the beginning of the apprehension of life, whether intellectual or physical, had sounded, and this planet, and all on it together, swung into their new course.

Cartwright's journalistic campaign, so elaborately prepared, was defeated before it began. It was true that the motions and movements with which the astronomers dealt were so small that some expert critics held that the whole result was doubtful, and could be accounted for by errors of observation. But such carping voices gained no hearing, for two reasons.

In the first place, the winter instead of coming in with a violent

wave of cold, began mildly and pleasantly, and in the second place a change in the thought of the people took place.

This most real and living space we know can be looked on as an instance of the laws of combination and permutation, and its properties can be deduced from an algebra, or it can be looked on and loved for its own sake. The mind can grasp it more and more fully in its direct complexity, knowing it at first hand as real even if removed from sense by its amplitude in unknown directions.

But who can say which is best? The magic of algebra or the love of space?

In Astria, at any rate, it happened that all the wielders of the combinations of one and two and three, and all the designers of all the possible configurations that could happen from abstract principles in this higher space, failed to detect any hang-hold of Farmer's theory on actual fact.

But amongst those who learned by means of models, making visible and tangible the aspects and views of the higher reality, were some who sprang, with a kind of inner awakening, to the knowledge of the third dimension.

And these, feeling rather than reasoning about that higher space, realized in their inner minds what these poor dwellers in a land of two dimensions could never see or touch—the actual nature of three-dimensional things moving and acting—things they could only apprehend in their plane of imagery as things succeeding things. And with this knowledge of inner sense, they brought to the examination of the minute and concealed processes which went on in nature a pattern and example which unlocked their secrets.

They found that many curious and inexplicable results, sphinx-like riddles of science, were the simplest and most to be expected consequences of the motions they inwardly realized. We see very plainly that when the particles considered are small enough, even in Astria, the movements must be three dimensional, for the things are real, and if small in a third dimension, are not vanishingly small. Thus the intimate knowledge of the third dimension was the key which unlocked the mystery of the minute. And so, guided marvellously amongst what some would consider gigantic delusions, vast mistakes, the good ship of Thought glided safe to port.

Farmer followed the work of the younger generation with interest, but also with a deepening sense of his own impotence. To himself he

seemed as a man denied the blessed spark of reason, for all the things he might have discovered in his long years of search were found by others, children to him in point of years. He left the busy city and the crowds of men and, half mortified, half amused but wholly glad for now the danger to the dear world was over, he devoted himself to his garden in far away Scythia. That rebellious and antagonistic mind forgot its struggles and vicissitudes in watching the little beads of verdure that sprang out of the dark earth.